ME 6125
Mechanics of Viscous Fluid

Lecture 3

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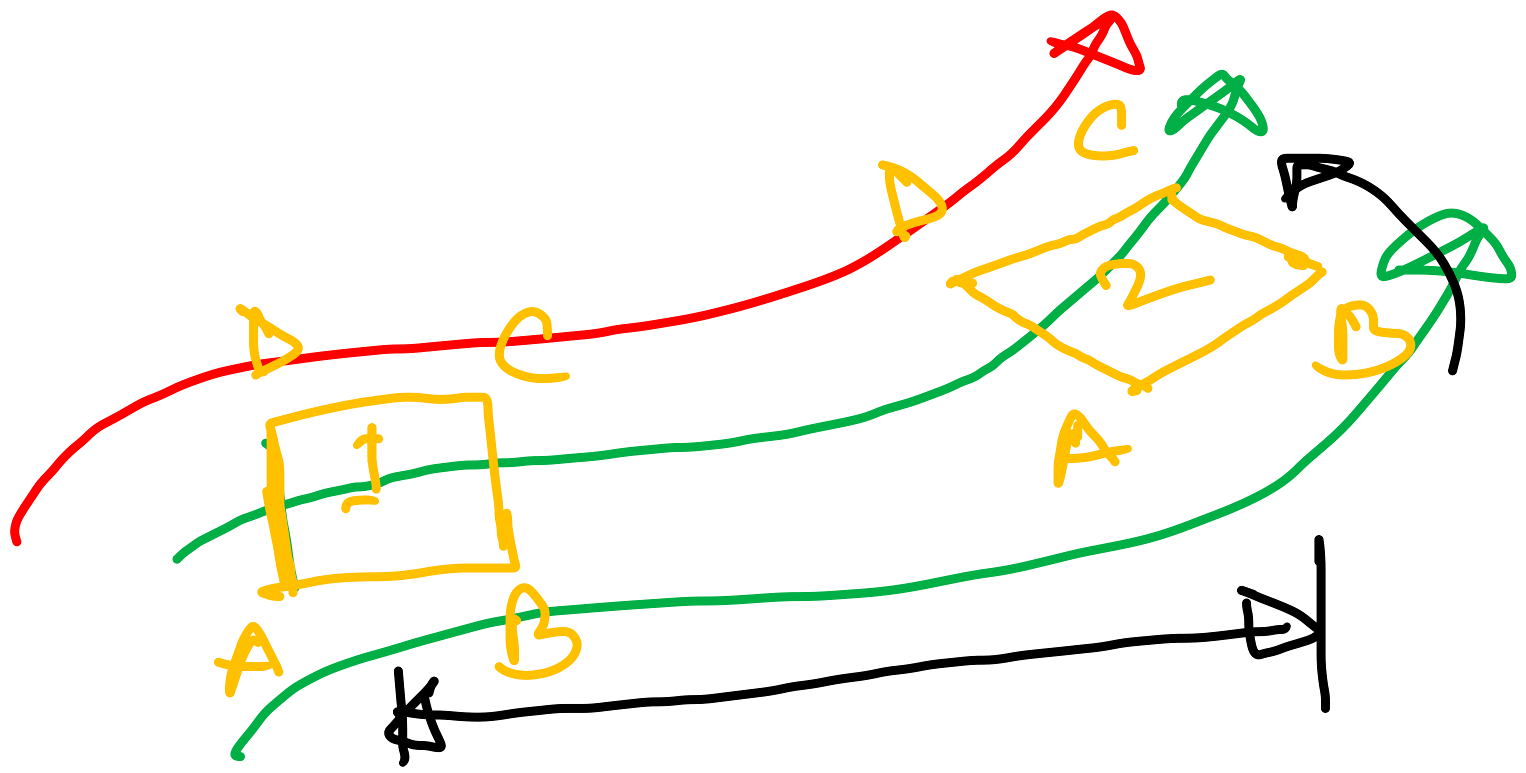
IMPACT Lab
Interfacial Multiscale Physics and Transport

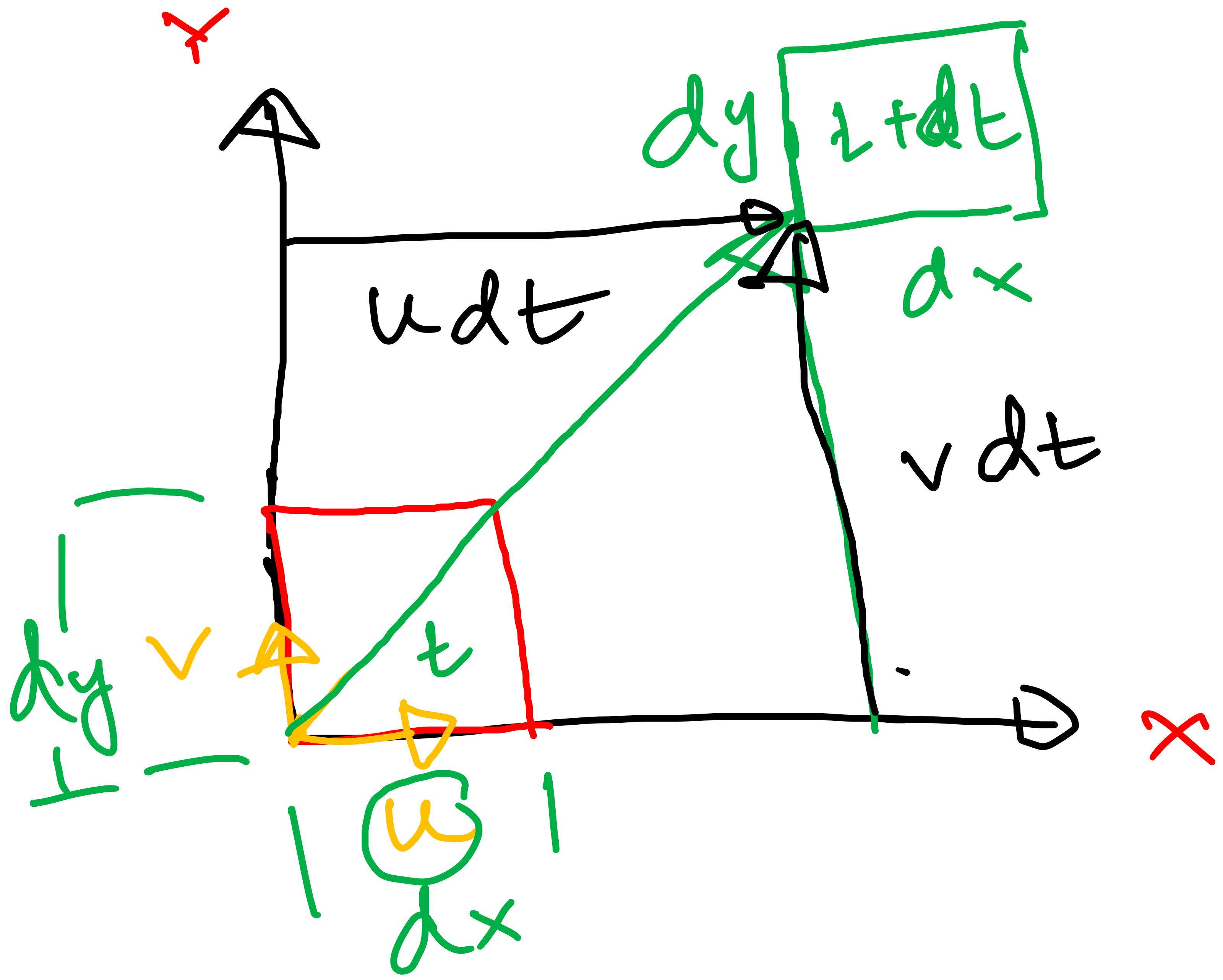
Kinematics

- study of motion
- Describing motion of fluids without discussing the forces.



Translation ✗ | Rotation ✗ → solid | Linear strain — solid | Shear strain —

Translation



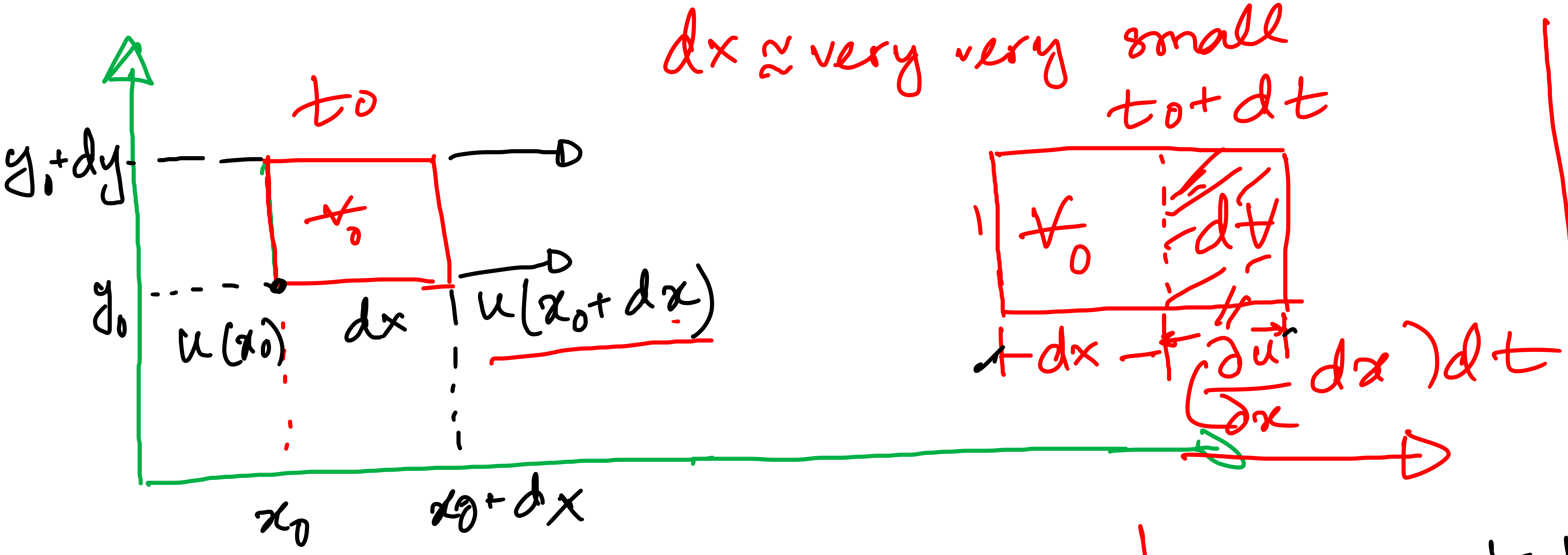
x- velocity = u
Y- velocity = v
z- velocity = w

Rate of Translation = u_i

$$\text{Translation}_x = u\,dt$$

$$\text{Rate of translation}_x = \underline{u}$$

Linear Strain Rate



Linear strain

$$= \frac{\text{new length} - \text{old length}}{\text{old length}}$$

Per Tylor series expension;

$$u(x_0+dx) \approx u(x_0) + \frac{\partial u}{\partial x} dx$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

* Linear strain

$$= \frac{\left[dx + \left(\frac{\partial u}{\partial x} dx\right) dt\right] - dx}{dx} = \frac{\left(\frac{\partial u}{\partial x} dx\right) dt}{dx}$$

$$= \frac{\partial u}{\partial x} dt$$

Linear strain rate $= \dfrac{\partial u}{\partial x}$

Divergence

$$d\forall = \left[\left(\frac{\partial u}{\partial x} dx\right) dt\right](dy\, dz) = \frac{\partial u}{\partial x} dt\, (dx\, dy\, dz)$$

$$= \frac{\partial u}{\partial x} dt\, \forall_0$$

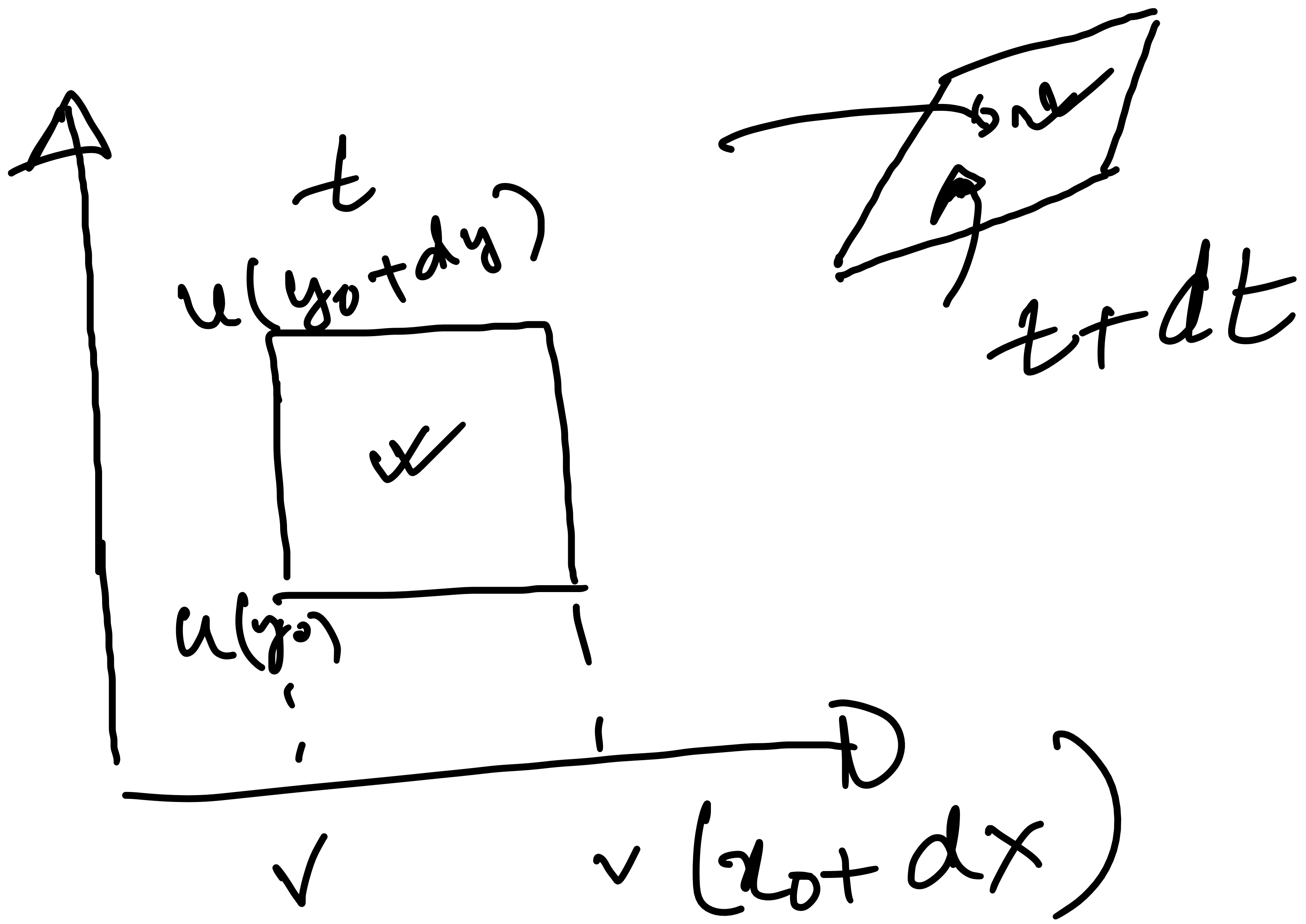
$$\Rightarrow \frac{\frac{d\forall}{V_0}}{dt} = \frac{\partial u}{\partial x} = \varepsilon_{xx}$$

→ linear strain rate describes the relative rate of change of volume of the fluid element.

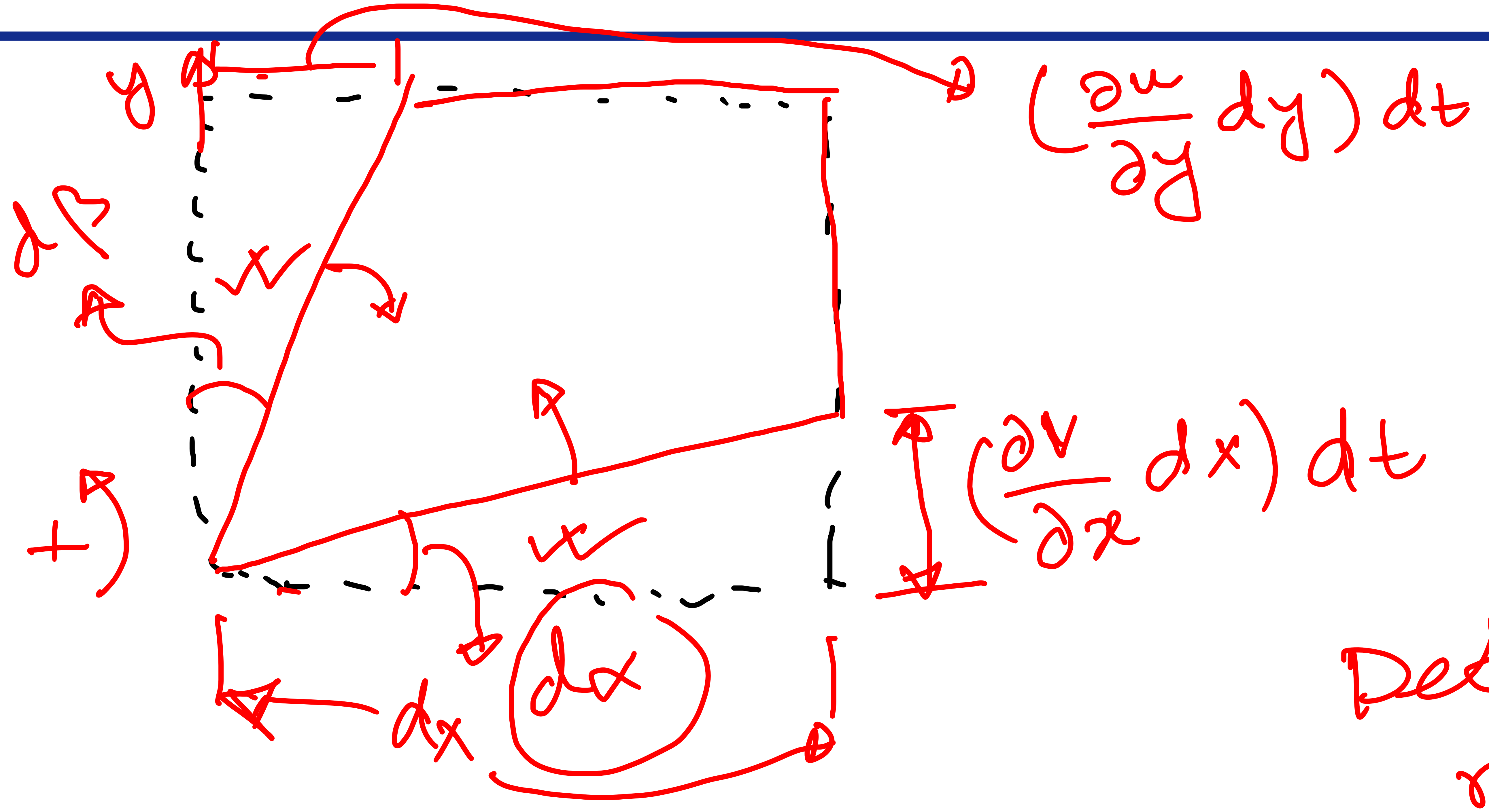
Total rate of change of fluid element volume $= \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \vec{V}$

for incompressible fluid, $\nabla \cdot \vec{V} = 0$ → why?

Angular strain rate



Deformation rate



$$\tan(d\alpha) = \frac{\left(\frac{\partial v}{\partial x} dx\right) dt}{dx}$$

$$\Rightarrow d\alpha = \left(\frac{\partial v}{\partial x}\right) dt$$

Deformation rate $= \dot{\gamma}_\alpha = +\frac{\partial v}{\partial x}$

$= \dot{\gamma}_\beta = -\frac{\partial u}{\partial y}$

Avg. Angular velocity, $\Omega_z = \frac{1}{2}(\dot{\gamma}_\alpha + \dot{\gamma}_\beta)$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Shear strain rate

Rotation and vortecity