ME 6125
Mechanics of Viscous Fluid

Lecture 4

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IMPACT Lab
Interfacial Multiscale Physics and Transport

Conservation of laws

mass conservation $\frac{dm_{sys}}{dt} = 0$

momentum " : —

Energy conse " —

system → Control volume

↓

close mass system

Reynolds Transport Theorem (RTT)-overview

- A system [also called a closed system] is a quantity of matter of fixed identity. No mass can cross a system boundary.

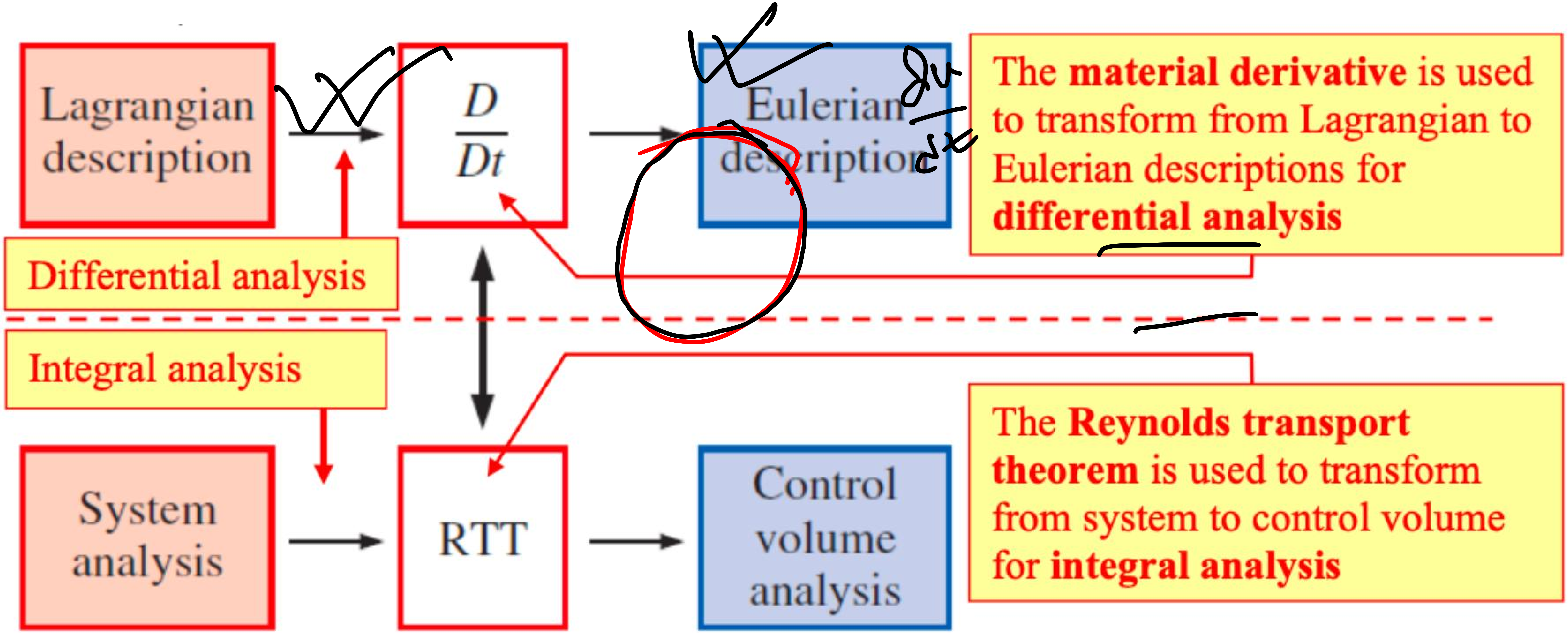
- A control volume [also called an open system] is a region in space chosen for study. Mass can cross a control surface (the surface of the control volume).

- The fundamental conservation laws (conservation of mass, energy, etc.) apply directly to systems.

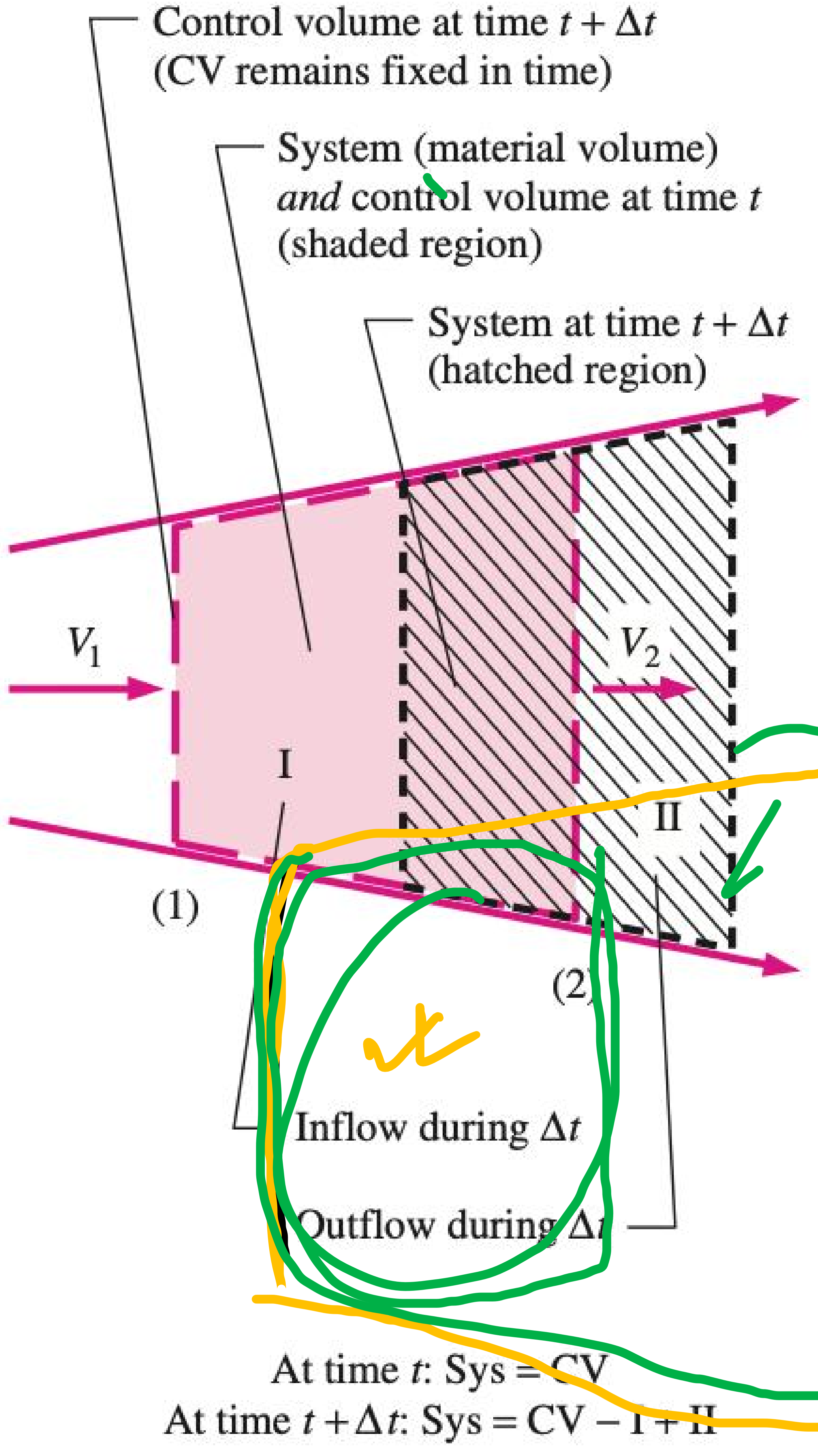
- However, in most fluid mechanics problems, control volume analysis is preferred over system analysis (for the same reason that the Eulerian description is usually preferred over the Lagrangian description).

- Therefore, we need to transform the conservation laws from a system to a control volume. This is accomplished with the Reynolds transport theorem (RTT).

Analogy Between the Material Derivative and the RTT



Reynolds Transport Theorem (RTT)



$$B_{\text{sys},\,t} = B_{\text{CV},\,t} \qquad \text{(the system and CV concide at time } t\text{)}$$

$$B_{\text{sys},\,t+\Delta t} = B_{\text{CV},\,t+\Delta t} - B_{\text{I},\,t+\Delta t} + B_{\text{II},\,t+\Delta t}$$

$$\frac{B_{\text{sys},\,t+\Delta t} - B_{\text{sys},\,t}}{\Delta t} = \frac{B_{\text{CV},\,t+\Delta t} - B_{\text{CV},\,t}}{\Delta t} - \frac{B_{\text{I},\,t+\Delta t}}{\Delta t} + \frac{B_{\text{II},\,t+\Delta t}}{\Delta t}$$

$$\boxed{\frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{CV}}}{dt} - \dot{B}_{\text{in}} + \dot{B}_{\text{out}}}$$

$t+\Delta t$

t

$$\boxed{\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho b \, dV + \int_{\text{CS}} \rho b \vec{V}\cdot\vec{n}\, dA}$$

Reynolds Transport Theorem (RTT)

$$\frac{d(\;)}{dt} = \frac{d}{dt}\int_{CV} \rho\,(\;)\,dV + \int_{CS} \rho\,(\;)$$

$$\boxed{\frac{dm}{dt} = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, \vec{V}\cdot d\vec{A}}$$

Q. There is a steady 2D flow of water (
a large outlet as sketched. width = 1m

Soln:

$$\frac{dB_{sys}}{dt} = \cancel{\frac{d}{dt}\int_{CV} \rho b \, dV}^{\,0} + \int_{CS} \rho b \, \vec{V} \cdot d\vec{A}$$

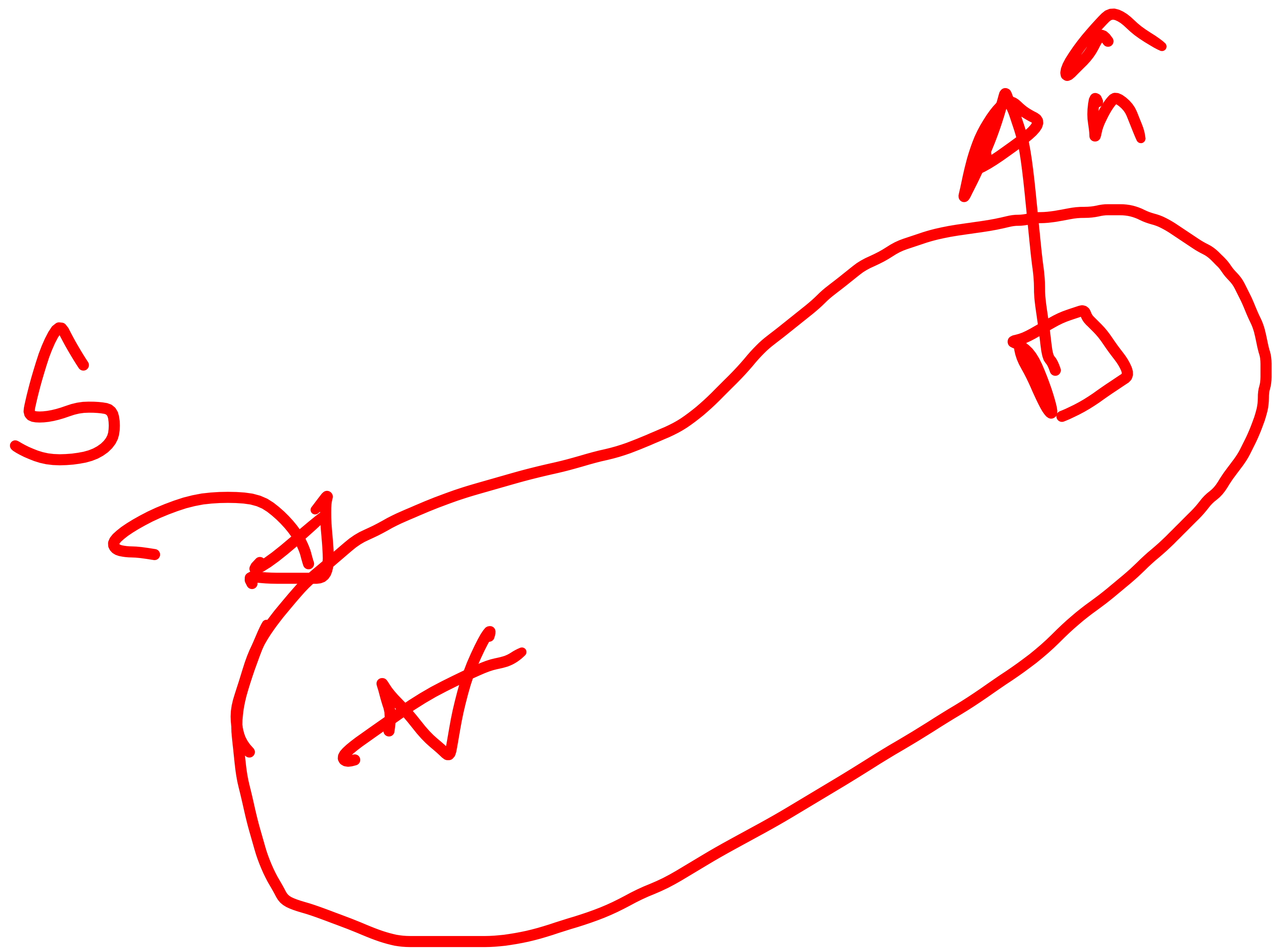
$$\vec{V} \cdot \hat{n} = \left[(3.22\hat{i} + 1.5\hat{j}) \cdot (\hat{i})\right]$$

$$= 3.22$$

$$\int \rho b \vec{V} \cdot \hat{n} \, dA = \rho \cancel{b}^{\,1} u s \int_{y_1}^{y_2} dA$$

Gauss Theorem

(Divergence Theorem



statement: The sum of the property in a closed volu[me] ... the flux of that proper[ty] bounding volume surface

symbolic:

$$\int_{\forall} \nabla \cdot \vec{F} \, d\forall = \oint \vec{F} \cdot \hat{n} \, dA$$

Leibniz theorem

How to differentiate an integral w.r.t. integrand and limits of integration of time.

1-D Leibniz theorem:

$$\frac{d}{dt}\int_{x=a(t)}^{x=b(t)} F(x,t)\,dx = \int_{a(t)}^{b(t)} \frac{\partial F}{\partial t}\,dx + \frac{\partial b}{\partial t} F$$

Example: $\dfrac{d}{dt}\int_0^{ct} e^{-x^2}\,dx$

$a(t) = 0$

$\dfrac{da}{dt} = 0$

$b(t) = ct$

$\dfrac{db}{dt} = c$

$F(x,t) = e^{-x^2}$

$\dfrac{dF}{dt} = 0$

$$\boxed{\frac{d}{dt}\int_0^{ct} e^{-x^2}\,dx = 0 + c\,e^{-b^2} - 0 = c\,e^{-b^2}}$$