# ME 6125
# Mechanics of Viscous Fluid

## Lecture 1

**Dr. Abdul Aziz Shuvo**
Assistant Professor
Department of Mechanical Engineering, BUET

Email: shuvo@me.buet.ac.bd

IMPACT Lab
Interfacial Multiscale Physics and Transport

# Recall

- RTT: $dB_{sys}/dt = d/dt \int_{cv} \rho\, b dV + \int_{cs} \rho\, b(V\, n) dA$

$b = \dfrac{B_{sys}}{m}$

- Gauss Theorem: The sum of the change of a property in a closed volume is equal to the flux of that property through the bounding volume. e.g. $\int_{cv} \nabla F dV = \int_{cs} (Fn) dA$

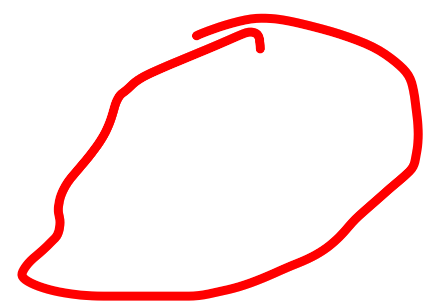
- General Leibniz Theorem:

$$d/dt \int_{cv} F\, dV = \int_{cv} d\, F/dt \; dV + \int_{cs} F\, (Vn) dA$$

$$\int_{cv} \frac{dF}{dt} dV + \int_{cs} F(\underline{V} \cdot \underline{\hat{n}})\, dA$$

# Conservation of mass

- Rate of change of mass of a material region is zero.



- $dM/dt = d/dt \int_v \rho \, dV = 0$

$$\frac{d}{dt}\int_v \rho \, dV = 0$$

- Bounding surface of material region (MR) is moving with the local velocity, $u_i$ . Then the COM is:

$$\boxed{\frac{d}{dt}\int_{mr} \rho \, dV = \int_{CV,mr} \frac{\partial}{\partial t} \rho \, dV + \int_{CS,mr} \rho (\bar{u} \cdot \bar{n}) \, dA = 0}$$

# Conservation of mass, cont'd

# $\frac{d}{dt}\int \rho \, dV = \int_{CV} \frac{\partial}{\partial t}\rho \, dV + \oint \rho u_i n \, dA = 0$ —— ①

Apply Gauss theorem: $\oint \rho u_i n \, dA = \int_{CV} \frac{\partial}{\partial x_i}(\rho u_i) \, dV$ —— ②

let sub. ② into ① $\Rightarrow$

$$\int_{CV} \frac{\partial}{\partial t}\rho \, dV + \int_{CV} \frac{\partial}{\partial x_i}(\rho u_i) \, dV = 0$$

$$\Rightarrow \int_{CV} \left[ \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}(\rho u_i) \right] dV = 0$$

# Conservation of mass, cont'd

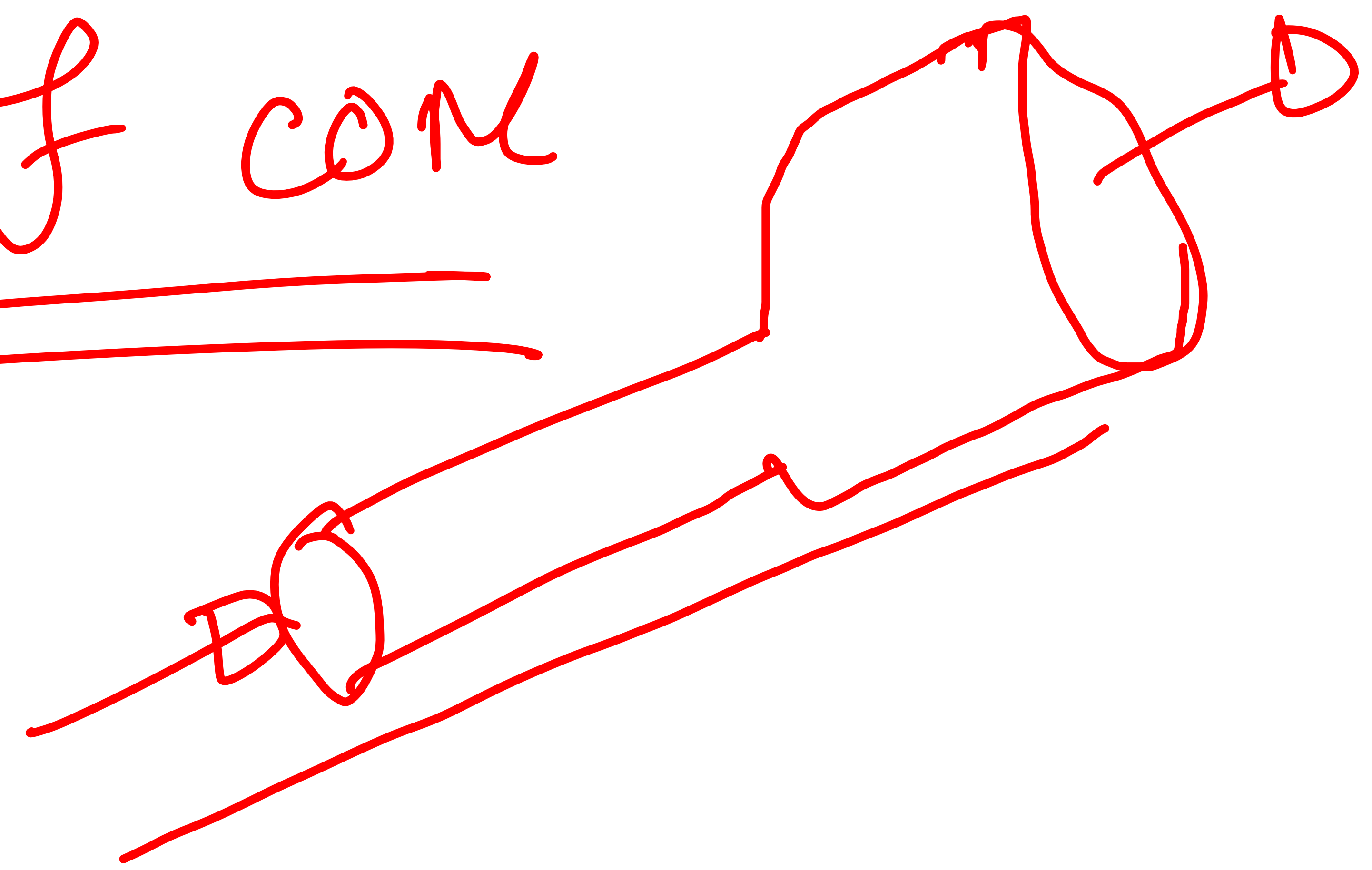
$$\int_{CV}\left[\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}(\rho u_i)\right]dV = 0 \Rightarrow \int_{CV}\left[\partial_0 \rho + \partial_i(\rho u_i)\right]dV = 0 \quad (3)$$

$$\partial_0 = \frac{\partial}{\partial t} \qquad \partial_i = \frac{\partial}{\partial x_i}$$

$\Rightarrow$ (3) is only zero when integrand $= 0$

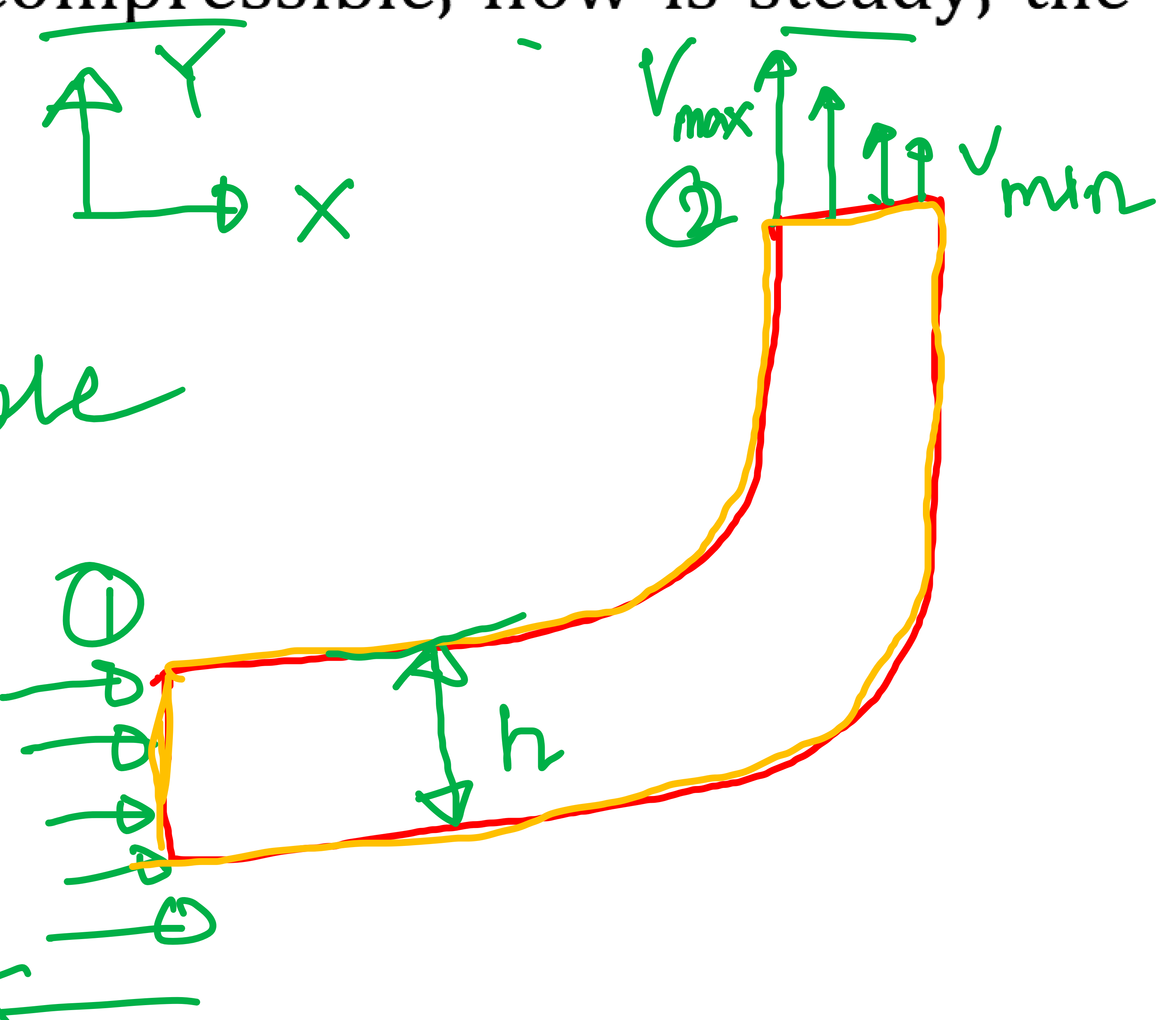
$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0}$$

Differential form of COM



# Example 1

- Consider water flowing in 2D channel depicted in the figured. The Channel depth is w ~~with~~ where w>>h. If the $V_{max} = 2V_{min}$, U=7.5 m/s and h=15.5 mm, find $V_{min}$. Assume that fluid is incompressible, flow is steady, the velocity ~~id~~ is uniform at the inlet and linear at the outlet.




Step 1: draw CV / coordinates

Step 2: Assumptions: ① incompressible ② steady
③ gravitation effect negligible

Step 3: Governing eqn's: $\cancel{\int_{CV} \frac{\partial \rho}{\partial t} d\forall}^{\,0} + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$ ①

$\Rightarrow \int_{CS} \vec{V} \cdot d\vec{A} = 0$ — ①

$V_{avg} = \frac{V_{max} + V_{min}}{2} = \frac{3}{2} V_{min}$

Surface 1: $-Uwh$

Surface 2: $+V_{avg}wh = \frac{3}{2} V_{min} wh$

① $\Rightarrow -Uwh + \frac{3}{2} V_{min} wh = 0 \Rightarrow \boxed{V_{min} = \frac{2}{3} U}$

Ans

# Conservation of linear momentum

- Time rate of change of linear momentum of a MR is sum of all forces on that region.
- What kind of forces, we have in body:
    - Surface force
    - Body force

* External forces

- Then COLm: $\sum F = \int \rho\, B_i dV + \int R_i\, dS$ — ①

Body force

Surface force

Rate of momentum

$$\frac{d}{dt}\int_{cv} \rho\, u_i\, d\forall \quad — ②$$

# Conservation of linear momentum, cont'd

# Surface force: $R_j = T_{ij}\, dA_i \rightarrow T_{ij} = -p\delta_{ij} + \tau_{ij}$

surface can be subdivided into pressure and viscous part

from ②
$$\frac{d}{dt}\int_{CV} \rho u_i \, d\forall = \int_{CV} \frac{\partial}{\partial t}(\rho u_i)\, d\forall + \underbrace{\int_{CS} \rho u_j u_i \, dA_j}_{\text{Apply Gauss}}$$

$$\int_{CV} \partial_j (\rho u_j u_i)\, dV$$

② $= 0$
$$\int_{CV} \left[\partial_0(\rho u_i) + \partial_j(\rho u_j u_i)\right] d\forall$$

# Conservation of linear momentum, cont'd

As per COLM ② = ①

$$\int_{CV} [\partial_0 (\rho u_i) + \partial_j (\rho u_j u_i)]\, d\forall = \int_{CV} \rho B_i\, d\forall + \int_{CS} R_i\, ds$$

$$\int (-p\delta_{ij} + \tau_{ij})\, dS$$

Gauss

$$\int_{CV} \partial_i (-p\delta_{ij} + \tau_{ij})\, d\forall$$

$$\Rightarrow \int_{CV} [\partial_0 (\rho u_i) + \partial_j (\rho u_j u_i) - \rho B_i + \partial_i (p\delta_{ij}) - \partial_i \tau_{ij}]\, d\forall = 0$$

③

# Conservation of linear momentum, cont'd

③ is zero if its integrand is zero: $\delta_{ij}$

$$\partial_0(\rho u_i) + \partial_j(\rho u_j u_i) - \rho B_i + \partial_i(P\delta_{ij}) - \partial_i \tau_{ij} = 0$$

$$\Rightarrow \boxed{\partial_0(\rho u_i) + \partial_j(\rho u_j u_i) = -\partial_i P\delta_{ij} + \partial_i \tau_{ij} + \rho B_i}$$

Differential COLM

Eq^n