ME 1102
Mechanical Engineering Fundamentals

FLUID MECHANICS

Fluid Mechanics

Fluid Mechanics is a branch of physics, concerned with the behavior of **liquids** or **gases** at rest (**Fluid Statics**) and motion (**Fluid Dynamics**).

It is a study which is applied to various natural phenomena, almost every discipline of engineering, biology and life sciences even in warfare.

Industrial Piping System

Steam Boiler

Fluids

A fluid is defined as a substance that deforms continuously under the action of shear stress of any magnitude. A shearing stress is a force per unit area that is created whenever a tangential force acts on a surface.

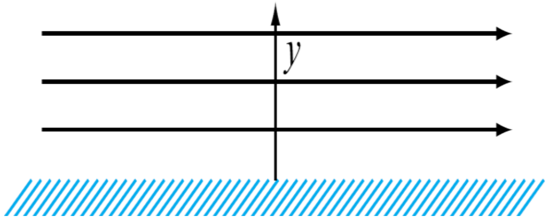
Consequently,

If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.

- Fluids are unable to retain any unsupported shape.
- Solids offer **permanent resistance** to a deforming force.
- Fluid at rest must be in zero shear stress often called ***hydrostatic stress*** condition in analyses.

Shear Stress in a Moving Fluid

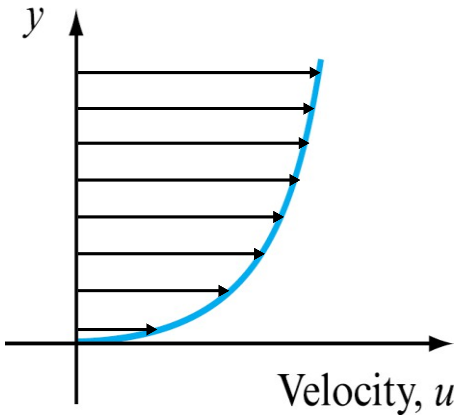
Shear stresses *are* developed when the fluid is in motion.
In motion, the particles of the **fluid move relative to each other** so that they have different velocities, causing the original shape of the fluid to become distorted.



On the other hand, if the velocity of the fluid is the same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Flow past a solid boundary are of great interest in engineering systems. (flow inside a pipe, flow past and airfoil, open channel flow etc.).

The fluid in contact with the solid boundary adheres to it, therefore, have the same velocity as the boundary. Relative velocity of the fluid particles in contact with the solid surface will be zero. This is often referred as **“No Slip Condition”.**



Velocity Profile

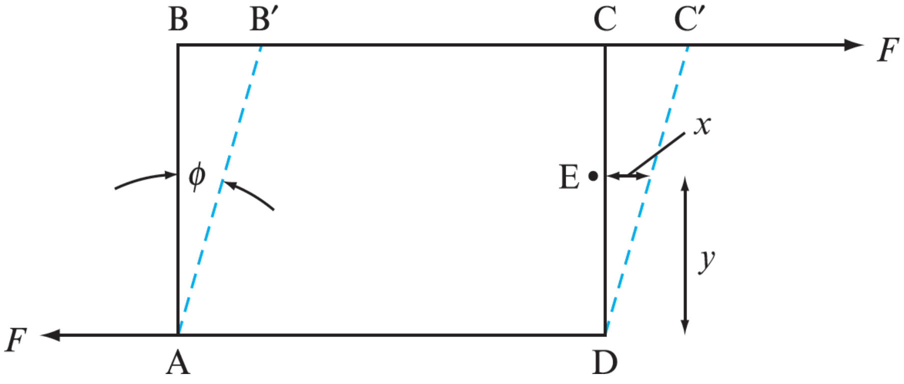
Shear Stress in a Moving Fluid

Let,
ABCD represents a fluid element, With,

Thickness = S (perpendicular to the diagram) Area, A = BC × S
Applied force = F (acting over the area, A)

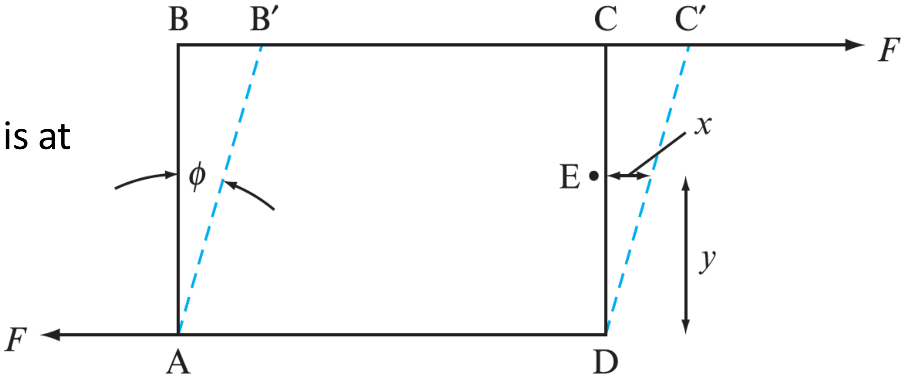
Shear stress, τ = $\frac{F}{A}$

Deformation (shear strain) = Angle, ϕ



- In a solid, ***φ* will be a fixed quantity** for a given value of τ, since a solid can resist shear stress within **elastic limit.**
- In a fluid, ***φ* will continue to increase with time** and the fluid will flow.
- It is found experimentally that, in a true fluid, the **rate of shear strain** (shear strain per unit time) is directly
  proportional to the shear stress.
- *Whereas,* In a solid, **shear strain** is proportional to the shear stress within the elastic limit (Hooke's Law).

Shear Stress in a Moving Fluid



Let,

At time t, a particle E (Fig.) moves through a distance x. E is at distance y from AD, then for small angles,

Shear strain, $\phi = \dfrac{x}{y}$ In differential form, $\phi = \dfrac{dx}{dy}$

Rate of shear strain = $\dfrac{d}{dt}\dfrac{dx}{dy} = \dfrac{d}{dy}\dfrac{dx}{dt} = \dfrac{du}{dy}$ $u = \dfrac{dx}{dt}$, velocity at point E

Assuming the experimental results (shear stress is proportional to shear strain rate),

τ = constant × $\dfrac{du}{dy}$, this constant is termed as ***dynamic viscosity, μ*** of fluid which varies from fluid to fluid.

Substituting,

$$\tau = \mu \frac{du}{dy}$$ **Newton's Law of Viscosity**

Viscosity

The most important property in the study of fluid flows.

- It can be considered as the internal stickiness of a fluid.
- It is one of the properties that controls the fluid flow rate in a pipeline.
- It accounts for the energy losses associated with the transport of fluids in ducts, channels, and pipes.
- It plays a primary role in the generation of turbulence.
- The rate of deformation of a fluid is directly linked to the viscosity of the fluid. For a given stress, a highly viscous fluid deforms at a slower rate than a fluid of lower viscosity.

Newton's law of viscosity, taking the direction of motion as the *x* direction and u= u(y) as the velocity of the fluid in the *x* direction at a distance *y* from the boundary, the shear stress in the *x* direction is given by-

$$\boldsymbol{\tau = \mu \frac{du}{dy}}$$

μ = Co-efficient of dynamic viscosity

The ***coefficient of dynamic viscosity μ*** can be defined as the shear force per unit area (shear stress τ) required to drag one layer of fluid with unit velocity past another layer a unit distance away from it in the fluid.

Viscosity

From Newton's Law:

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{Force}{Area}}{\frac{Velocity}{Distance}} = \frac{Force\ x\ Time}{Area} = \frac{Mass}{Length\ x\ Time}$$

Unit: $Nm^{-2}s$ / $Pa.s$ / $kgm^{-1}s^{-1}$, often measured in Poise. **1 Poise = 10^{-1} $Nm^{-2}s$**

Dimension: $ML^{-1}T^{-1}$

Typical values:

Water: 1.14×10^{-3} $Nm^{-2}s$

Air: 1.85×10^{-5} $Nm^{-2}s$ at 25 ^oC

Iron: 6.7×10^{-3} $Nm^{-2}s$ (at 1550 ^oC), 5.6×10^{-3} $Nm^{-2}s$ (at 1700 ^oC), 5.2×10^{-3} $Nm^{-2}s$ (at 1850 ^oC)

The ***Kinematic Viscosity,*** ν is defined as the ratio of dynamic viscosity to mass density:

$$\vartheta = \frac{\mu}{\rho}$$

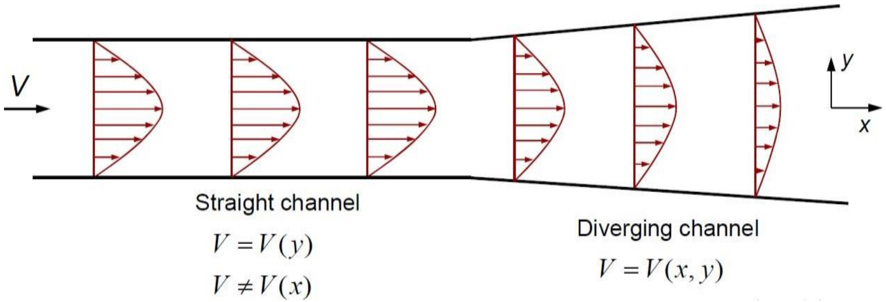
Unit: m^2/s (SI unit) often measured in stokes(St), 1 m^2/s = 10^4 St. Dimension: L^2T^{-1}

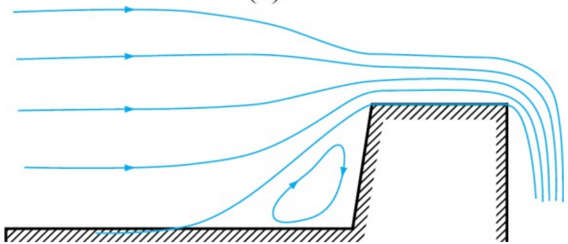
Fluid Flow

❖ **One-, Two-, and Three-Dimensional Flows**

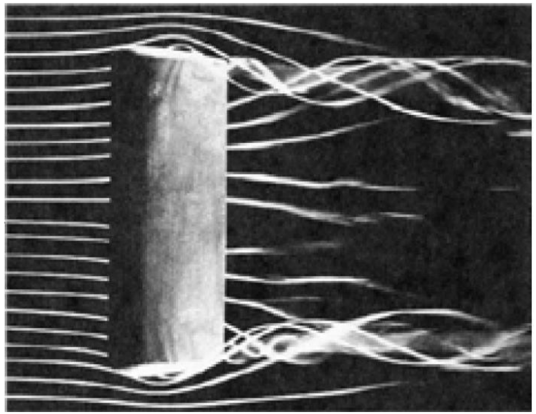
Generally, a fluid flow is a rather complex three-dimensional, time dependent phenomena- $\boldsymbol{V} = \boldsymbol{V}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t})$. In many situations, however, it is possible to make simplifying assumptions that allow a much easier understanding of the problem.

1-D flow: $\boldsymbol{V} = \boldsymbol{V}(x)$ 2-D flow: $\boldsymbol{V} = \boldsymbol{V}(x, y)$ 3-D flow: $\boldsymbol{V} = \boldsymbol{V}(x, y, z)$





2-D Flow over a weir

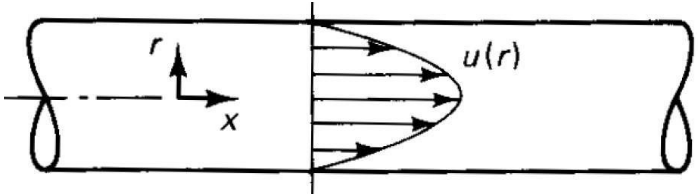


Complex 3-D Flow over a Model Wing

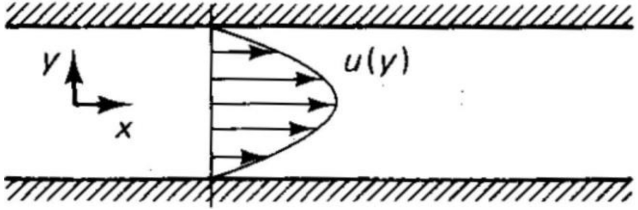
Fluid Flow

❖ One Dimensional Flow

Flow is described as ***one-dimensional*** if the factors, or parameters, such as velocity, pressure and elevation, describing the flow at a given instant, vary only along the direction of flow and not across the cross-section at any point. Such flows occur in long straight pipes or between parallel plates, as shown in Fig.



Flow in a Pipe



Flow between parallel plates

The velocity in the pipe varies only with **r**, i.e. ***u = u(r).***

The velocity between parallel plates varies only with the coordinate **y** , i.e. ***u = u(y).***

If the flow is unsteady, flow parameters may vary with time, ***i.e. u = u(r,t) and u = u(y,t).***

The one dimension is usually taken as the distance along the central streamline of the flow, even though this may be a curve in space, and the values of velocity, pressure and elevation at each point along this streamline will be the average values across a section normal to the streamline.

Fluid Flow

❖ **Uniform Flow and Steady Flow**

Flow is described as ***uniform flow*** if the velocity at a given instant is the same in magnitude and direction at every point in the fluid.

If, at the given instant, the velocity changes from point to point, the flow is described as ***non-uniform flow.***

In practice, when a fluid flows past a solid boundary there will be variations of velocity in the region close to the boundary. However, if the size and shape of the cross-section of the stream of fluid are constant, the flow is considered to be uniform.

A ***steady flow*** is one in which the velocity, pressure and cross-section of the stream may vary from point to point but do not change with time.

If, at a given point, conditions do change with time, the flow is described as ***unsteady flow or transient flow.***

In practice, there will always be slight variations of velocity and pressure, but, if the average values are constant, the flow is considered to be steady.

Fluid Flow

There are, therefore, four possible types of flow:

- ***Steady uniform flow:***

Conditions do not change with position or time. The velocity and cross-sectional area of the stream of fluid are the same at each cross-section. Example: flow of a liquid through a pipe of uniform bore running completely full at constant velocity.

- ***Steady non-uniform flow:***

Conditions change from point to point but not with time. The velocity and cross-sectional area of the stream may vary from cross-section to cross-section, but, for each cross-section, they will not vary with time: for example, flow of a liquid at a constant rate through a tapering pipe running completely full.

- ***Unsteady uniform flow:***

At a given instant of time the velocity at every point is the same, but this velocity will change with time: for example, accelerating flow of a liquid through a pipe of uniform bore running full, such as would occur when a pump is started up.

- ***Unsteady non-uniform flow:***

The cross-sectional area and velocity vary from point to point and also change with time: for example, a wave travelling along a channel.

Fluid Flow

- **Incompressible Flow**

An ***incompressible flow*** exists if the density of each fluid particle remains relatively constant as it moves through the flow field. That is,

Example: Liquid flows, low-speed gas flows, such as the atmospheric flow, are also considered to be incompressible flows.

- **Compressible Flow**

Flows are considered to be ***compressible*** if there is a significant change in density.
In case of compressible flows, the flowing fluid (commonly gas/air) can experience considerable **density changes as a result of externally applied pressures**.

In fluid mechanics, the effect of compressibility in the flow field can be assessed by a number called the "**Mach number**".
This dimensionless number is defined as- $M = \frac{V}{a}$, Where, V = local flow velocity, a = local speed of sound.

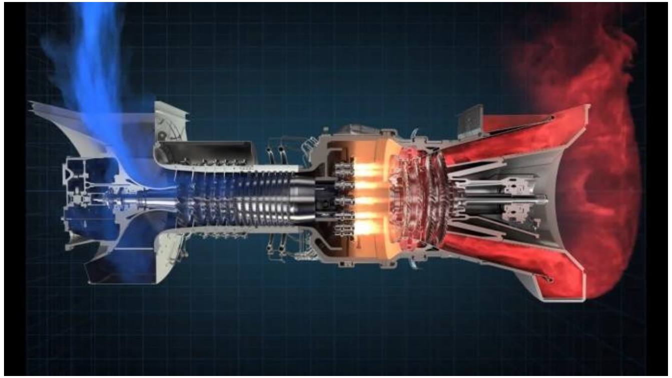
Fluid Flow

- **Compressible Flow**

If $M < 0.3$, density variations are at most 3% and the flow is assumed to be incompressible for standard air; this corresponds to a velocity below about 100 m/s or 300 ft/sec.

If $M > 0.3$, the density variations influences the flow and compressibility effects should be accounted for; such flows are **compressible flows**.

Compressible flows include the aerodynamics of high-speed aircraft, airflow through jet engines, steam flow through the turbine in a power plant, airflow in a compressor, and the flow of the air-gas mixture in an internal combustion engine.



Fluid Flow

❖ **Internal Flow and External Flow**

Flows are called ***internal flow*** when the flow exists interior to bodies.
<u>Example:</u> Flow in a circular pipe, duct flow, flow in a nozzle,human blood flow through arteries etc.

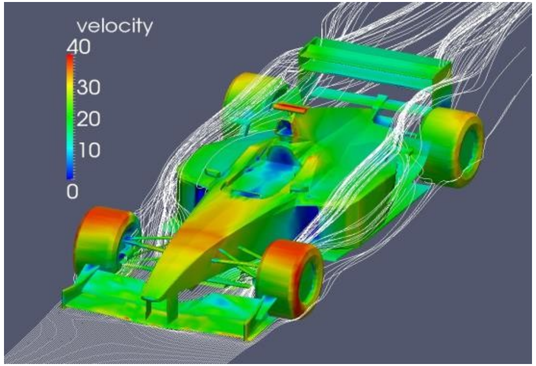
Flows are called ***external flow*** when the flow exists exterior to bodies.
<u>Example:</u> Flow past an airfoil, flow past a circular cylinder, flow around a car etc.

Both internal and external flows can be inviscid/ viscous; steady/unsteady; uniform/ non-uniform; compressible/ incompressible.

They both can be treated as 1D/2D or 3D flow.

Viscous effect such as boundary layer regions are created in both of flows.

Effect of viscosity causes ***drag in external flows*** *(power consumption of an airplane increases)* and causes various kind of ***losses in internal flows*** (*losses in pipes, pumping power consumption increases)*

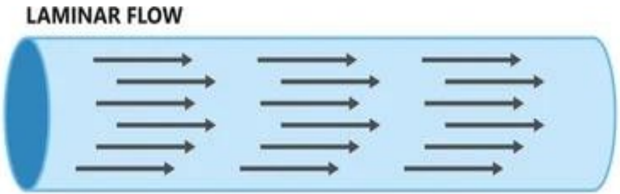


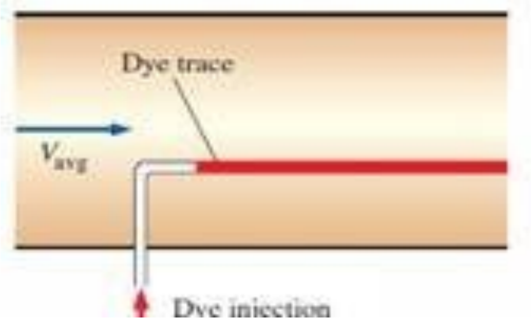
Fluid Flow

❖ Laminar Flow and Turbulent Flow

All the flow phenomena discussed before can be laminar or turbulent.

Laminar flow occurs when a fluid flows in ***parallel layers*** and slide past one another. This flow is very regular and smooth. There will be no lateral mixing and interaction between the layers. Laminar flow occurs at low speed.



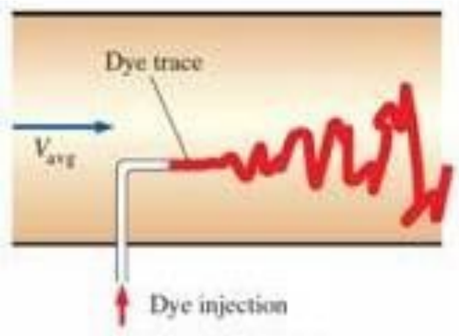


Fluid Flow

❖ **Laminar Flow and Turbulent Flow**

Turbulent flow is a fluid motion with particle trajectories varying randomly in time, in which irregular fluctuations of various flow properties arise.

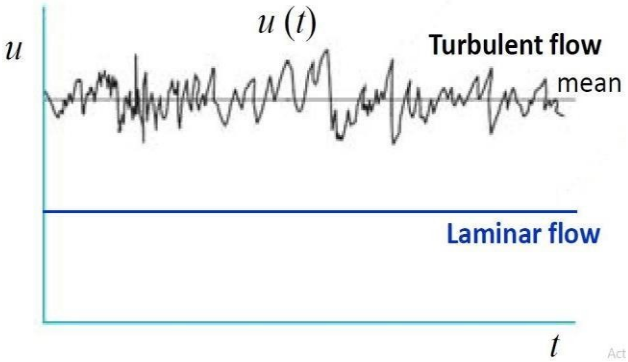
Due to conditions imposed by the geometry and flow field, such as surface topography, surface roughness, surface mass injection (or suction), pressure gradient, surface temperature and so on, the interaction of the fluid particles increases and takes place at the macroscopic level; the **streamlines of the flow field are no longer well-behaved**. This type of flow is known as ***turbulent flow***.

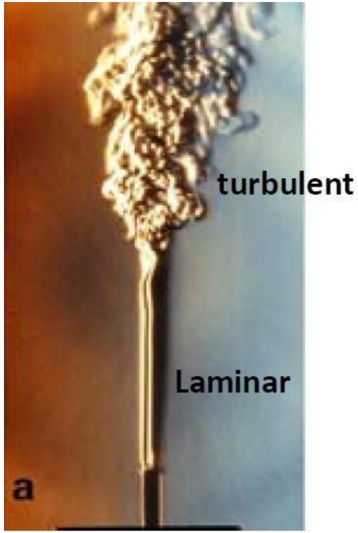


Fluid Flow

The fundamental difference between laminar and **turbulent flow** lies in the **chaotic, random behavior of the various flow properties**. Such variations occur in the three components of velocity, the pressure, the shear stress, the temperature, and any other variable that has a field description (such as density).

A typical time trace of the axial component of velocity measured at a location in the flow is shown in the figure. **Its irregular, random nature is the distinguishing feature of turbulent flow.**





Candle plume

The nature of the flow (laminar / turbulent) can be characterized based of one dimensionless number called the "**Reynolds number, Re**".

$$Re = \frac{\rho V L}{\mu}$$

Where, ρ = Density of fluid, **V** = Velocity of flow, μ = Dynamic Viscosity, **L** = Characteristic Length

Characteristic length, L varies from flow to flow (for a pipe flow L = diameter of the pipe, for flow past an airfoil (L = airfoil chord etc.)

Fluid Flow

The nature of the flow (laminar/turbulent) can be characterized based on one dimensionless number called the "**Reynolds number, Re**". Re is defined by:

$$\mathrm{Re} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$$

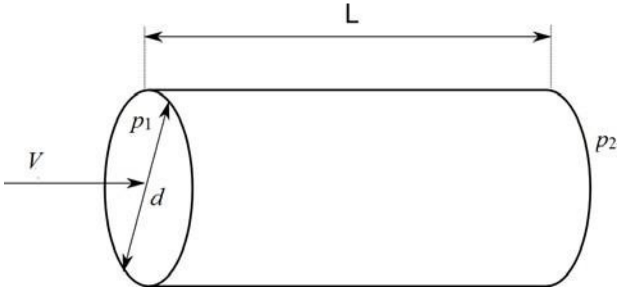
This number (Re) represents the relative importance of flow velocity (inertia) to the frictional (viscous) characteristics of fluid. This is the single most important number to be considered in any fluid-mechanical system being considered (for example: pipe flows, boundary-layer flows, nozzle flows, airfoil flows, lubrication system, biological flows, biofluidics, micro flows, etc.)

As general criteria:

For flow through smooth pipe:

$Re_d < 2300$; flow is laminar

$Re_d > 4000$; flow is turbulent

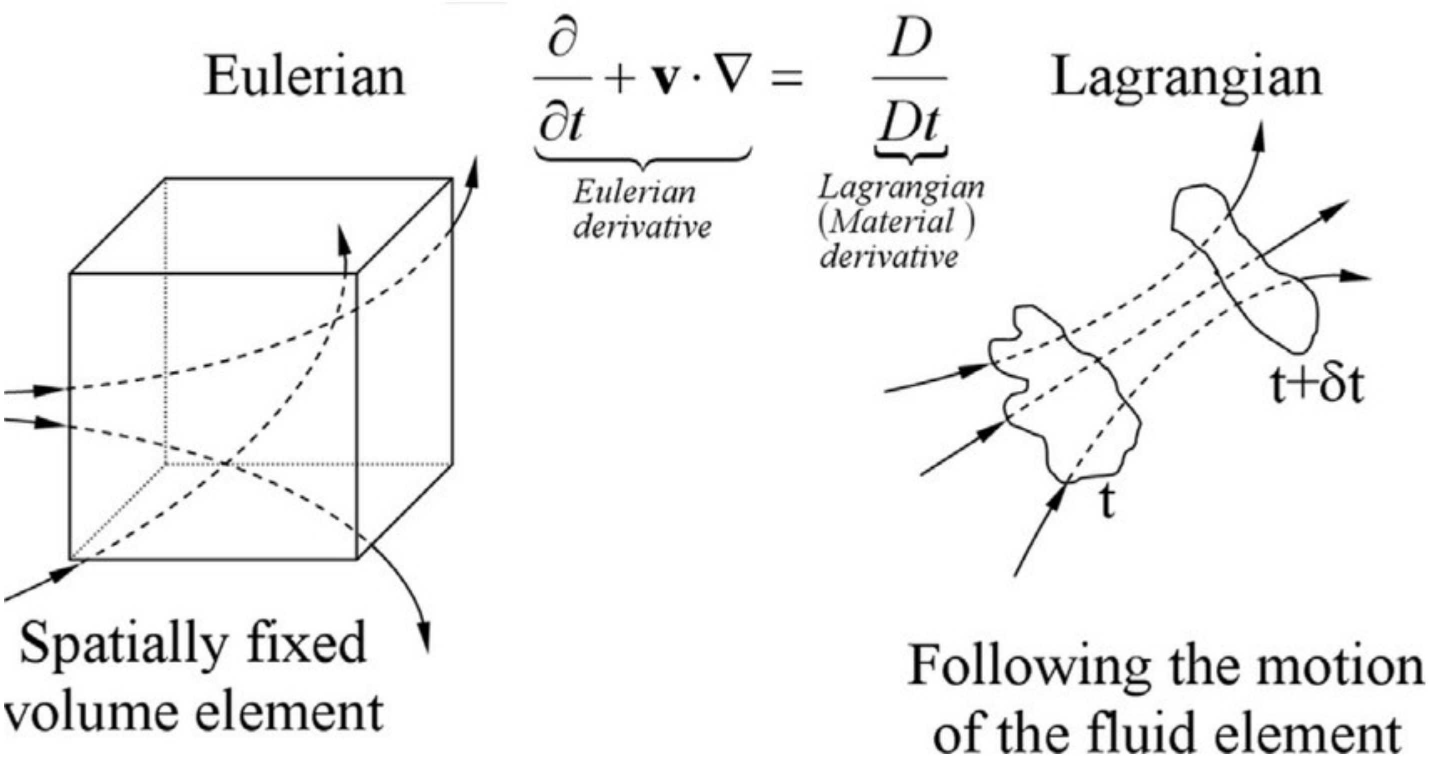


Description of Fluid Motion

- In the description of a flow field, it is convenient to think of individual particles each of which is considered to be a small mass of fluid, consisting of a large number of molecules, that occupies a small volume that moves with the flow.
- If the fluid is incompressible, the volume does not change in magnitude but may deform. If the fluid is compressible, as the volume deforms, it also changes its magnitude. In both cases the particles are considered to move through a flow field as an entity.
- There are two approaches of mathematical description of the physical quantities of a fluid motion as a function of space and time coordinates :

    - **Lagrangian Descriptions of Motion**
    - **Eulerian Descriptions of Motion**

Lagrangian and Eulerian Descriptions



Lagrangian and Eulerian Descriptions

- In the study of particle mechanics, where attention is focused on individual particles, motion is observed as a function of time. The position, velocity, and acceleration of each particle are listed as $\mathbf{s}(x_0, y_0, z_0, t)$, $\mathbf{V}(x_0, y_0, z_0, t)$, and $\mathbf{a}(x_0, y_0, z_0, t)$, and quantities of interest can be calculated.

- The point (x_0, y_0, z_0) locates the starting point of each particle. This is the Lagrangian description, named after Joseph L. Lagrange (1736–1813), of motion that is used in a course on dynamics.

- In the Lagrangian description many particles can be followed and their influence on one another noted. This becomes, however, a difficult task as the number of particles becomes extremely large in even the simplest fluid flow.

Eulerian Descriptions of Motion

- The region of flow that is being considered is called a flow field.
- In Eulerian description, points in a flow field is identified in space and then the velocity of fluid particles passing each point is observed; we can observe the rate of change of velocity as the particles pass each point, that is,

$$\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}$$

- We observe if the velocity is changing with time at each particular point, that is,

$$\frac{\partial V}{\partial t}$$

- In this Eulerian description, named after Leonhard Euler (1707- 1783), of motion, the flow properties, such as velocity, are functions of both space and time.
- In Cartesian coordinates the velocity is expressed as **V** = **V**(*x*, *y*, *z*, *t*).

Analysis of Fluid Dynamics

- When the flow characteristics of flowing fluid are functions of only any one of the three co-ordinate directions and time t, that is, the flow characteristics are functions of, say, x direction and time t, then it is called a one –dimensional flow. In such a case, the flow characteristics do not change in y and z directions. Further, if any of these flow characteristics does not change with time, it will be steady one-dimensional flow.
- Fluid flow is governed by three basic principles: conservation of mass, conservation of momentum, and conservation of energy
- We can mathematically formulate these principles of the fluid flow into corresponding governing equations as continuity equation, momentum equation, and energy equation, respectively.

Analysis of Fluid Dynamics

In analytical fluid dynamics, there are two approaches:

(1) Detail description of flow pattern at every point (x, y, z) in the flow field

– known as differential analysis (differential relations, differential equations)

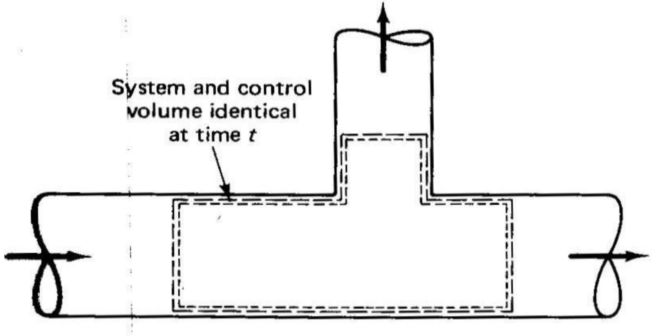
(2) Working with a finite region (control volume (CV)), making a balance of flow-in versus flow-out, and determining the **gross flow effects** such as the force or torque on a body or the total energy exchange.

– known as integral analysis (integral relations, integral equations)

Continuity Equation

- In a control volume law of conservation of mass states that the net mass flow through the volume will equal the mass stored or removed from the volume.
- The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.
- This principle of conservation of mass can be applied to a flowing fluid. Considering any fixed region in the flow constituting a control volume,

$$\left(\begin{array}{c}\textit{Total mass entering}\\ \textit{the CV during } \Delta t\end{array}\right) - \left(\begin{array}{c}\textit{Total mass leaving}\\ \textit{the CV during } \Delta t\end{array}\right) = \left(\begin{array}{c}\textit{Net change in mass within the CV}\\ \textit{during } \Delta t\end{array}\right)$$



Continuity Equation

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{\partial}{\partial t} m_{\text{element}}$$

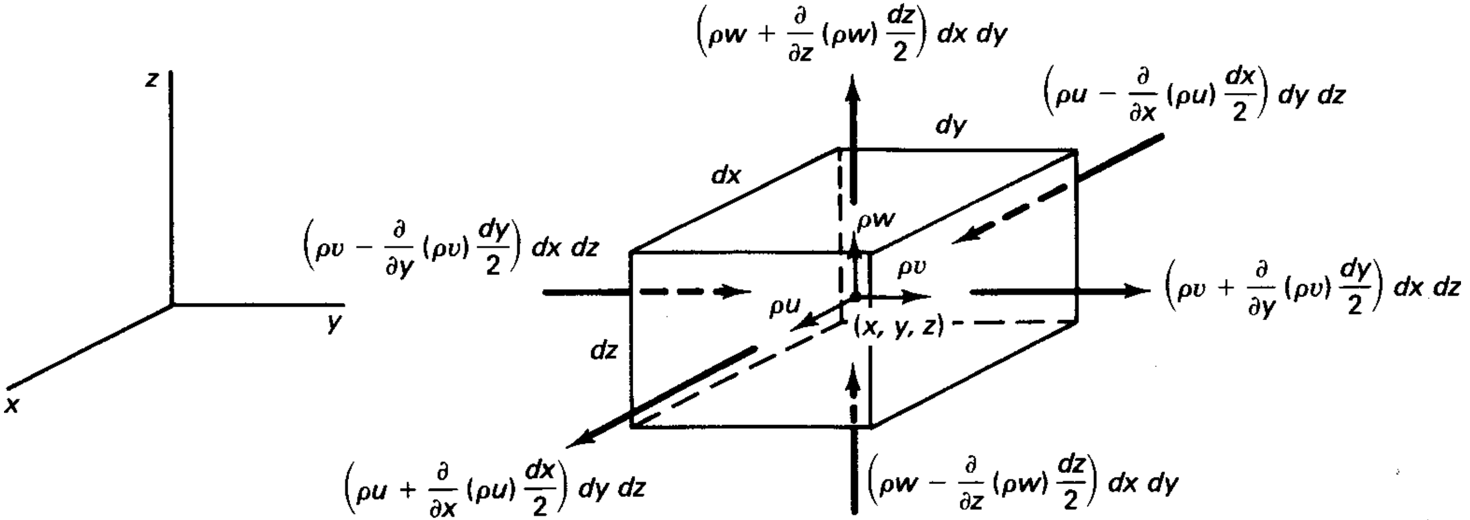
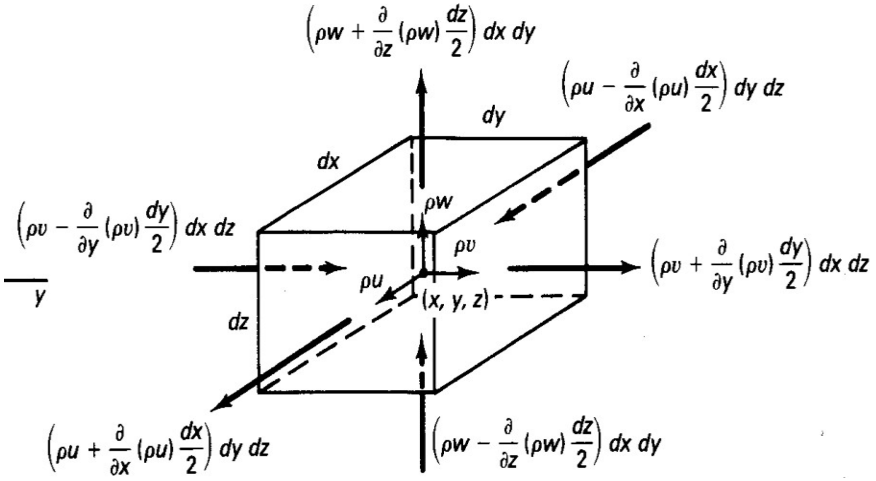


Figure 5.1 Infinitesimal control volume using rectangular coordinates.

Continuity Equation

- The rate of mass flow through the surfaces of the element can be obtained by considering the flow in each of the coordinate directions separately.
- To perform this mass balance, consider the mass rate of flow per unit area ρu, ρv, and ρw at the center of the element and then treat each of these quantities as a single variable.



X Direction:

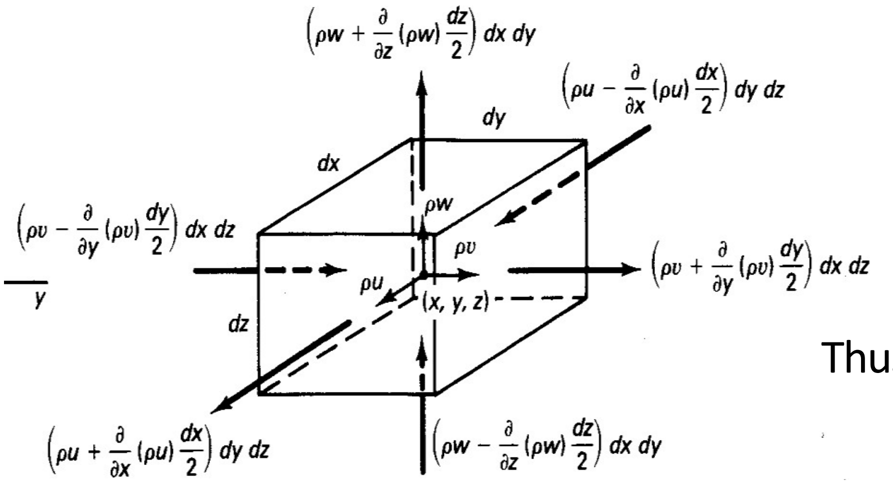
Mass flow at Rear Face: $(\rho u - \frac{\partial (\rho u)}{\partial x}\frac{\delta x}{2})\delta y \delta z$

Mass flow at Front Face: $(\rho u + \frac{\partial (\rho u)}{\partial x}\frac{\delta x}{2})\delta y \delta z$

Net rate of mass outflow: $-2\left(\frac{\partial (\rho u)}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$

$$= -\frac{\partial (\rho u)}{\partial x}\delta x \delta y \delta z$$

Continuity Equation



Similarly,

For y direction: $-\dfrac{\partial(\rho v)}{\partial y}\delta x\delta y\delta z$

For z direction: $-\dfrac{\partial(\rho w)}{\partial z}\delta x\delta y\delta z$

Thus, $the\ net\ rate\ of\ mass\ outflow$

$$\left(\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right)\delta x\delta y\delta z$$

- Since, we considered

$$the\ net\ mass\ transfer = mass\ inflow - mass\ outflow$$

$$\frac{\partial}{\partial t}(\rho\delta x\delta y\delta z) = -\left(\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right)\delta x\delta y\delta z$$

Continuity Equation

- Dividing both sides by $\delta x \delta y \delta z$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

- This equation is known as " Continuity Equation"

- Since density is considered a variable, we differentiate the products and put the above eqn in the form

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

- or, in terms of the substantial derivative

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

Continuity Equation

- The continuity equation is one of the fundamental equations of fluid mechanics and is valid for steady or unsteady flow, and compressible or incompressible fluids.

- For steady flow of compressible fluids:

$$\frac{\partial}{\partial t} = 0 \longrightarrow \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

- It states that the total rate of mass entering a control volume is equal to the total rate of mass leaving it.

- For Incompressible Fluids (Both Steady & Unsteady) The fluid density, $\rho = constant$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = 0$$

- The equation becomes,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Continuity Equation

- For 2D steady incompressible flow, $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- For 1D steady incompressible flow, $\rho u A = constant$

- Many engineering devices such as nozzles, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet). For these cases, Then Eq. reduces, for single-stream steady-flow systems, to

$$\rho_{in} V_{in} A_{in} = \rho_{out} V_{ou} A_{out}$$

$$or, V_{in} A_{in} = V_{ou} A_{out} \quad (Incompressible\ flow)$$

Problem 1

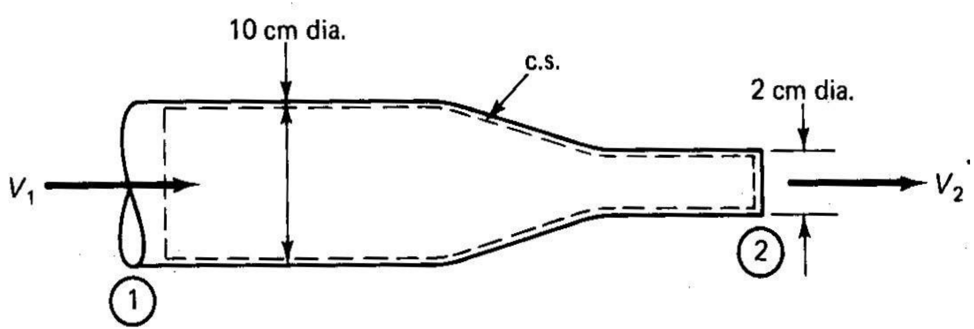
- The velocity distribution for the flow of an incompressible fluid is given by

  υ_x = 3 − x, υ_y = 4 + 2y, υ_z = 2 − z.

  Show that this satisfies the requirements of the continuity equation.

Problem 2

Water flows at a uniform velocity of 3 m/s into a nozzle that reduces the diameter from 10 cm to 2 cm. Calculate the water's velocity leaving the nozzle and the flow rate.



Solution

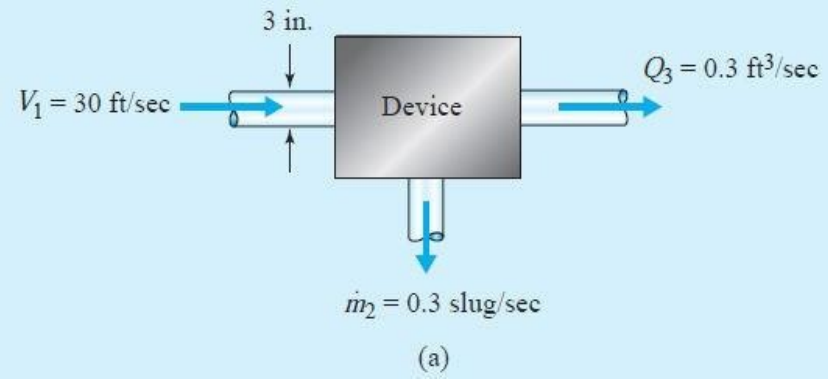
The control volume is selected to be the inside of the nozzle as shown. Flow enters the control volume at section 1 and leaves at section 2. The simplified continuity equation (is used since the density of water is assumed constant and the velocity profiles are uniform:

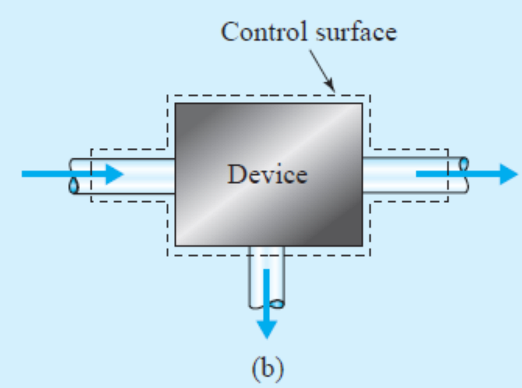
$$A_1 V_1 = A_2 V_2 \qquad \therefore \quad V_2 = V_1 \frac{A_1}{A_2} = 3 \frac{\pi \times 0.1^2/4}{\pi \times 0.02^2/4} = 75 \text{ m/s}$$

The flow rate or discharge is found to be: $Q = V_1 A_1 = 3 \times \pi \times 0.1^2/4 = 0.0236 \text{ m}^3/\text{s}$

Problem 3

Water flows in and out of a device as shown in Fig. E4.2a. Calculate the rate of change of the mass of water (dm/dt) in the device.





$$0 = \frac{dm}{dt} - \rho_1 A_1 V_1 + \dot{m}_2 + \rho_3 Q_3$$

Navier-Stokes equations

In an **incompressible flow** the density remains constant or the change in density is very negligible. In an incompressible flow of a **Newtonian Fluid** with constant viscosity, the momentum equations (derived from Newton's second law of motion) are:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho \frac{du}{dt}$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho \frac{dv}{dt}$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho \frac{dw}{dt}$$

These are the called **Navier-Stokes equations**, named after C. I. M. H. Navier (l785-1836) and Sir George G. Stokes (1819-1903), who are credited with their derivation. These are second-order nonlinear partial differential equations.

Navier-Stokes equations

- In vector notations, the **Navier-Stokes equations** may be written as:

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V}$$

- Which may be interpreted as:
- **Inertia force** per unit vol. = **gravity force** per unit vol. + **pressure force** per unit vol. + **viscous force** per unit volume
- There are four unknowns: *p*, *u*, *v*, and *w*. So, for solution, it should be combined with the incompressible continuity relation to form four equations in these four unknowns.
- The continuity equation for incompressible flow in steady state is:

$$\nabla \cdot \mathbf{V} = 0 \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible Inviscid flow....

- **Computation Fluid Dynamics** (CFD) is the branch of fluid mechanics that uses numerical methods to solve the **Navier-Stokes equations** and **continuity equation** with appropriate boundary conditions and without any analytical simplification.

- However, analytical solution for many **viscous flow** problems are also possible through some simplifying assumptions.

- If viscosity is assumed to be zero (**inviscid flow approximation**), the second order terms in the **Navier-Stokes equations** are lost and we get the **Euler's Equation,** as:

$$\rho \frac{d\mathbf{V}}{dt} = \rho\mathbf{g} - \nabla p$$

- Integration of this equation along a streamline results in **Bernoulli's equation**.

Reference Books

Text Book (s):

- M.C. Potter, D.C. Wiggert, Mechanics of Fluids, 3rd Edition, 2010, ISBN: 978-0-495-43857-1.
- F. M. White, Fluid Mechanics, 7th Edition, 2011, ISBN: 978-007-131121-2.

Reference books: (for further reading)

i. Munson, Okiishi, Huebsch, Rothmayer, Fundamentals of Fluid Mechanics, 7th Edition, 2013, ISBN: 978-1-118-18676.

ii. Fox and McDonald, Introduction to Fluid Mechanics, 9th Edition, 2015, ISBN: 978-1118912652.

iii. J. F. Douglas, J. M. Gasiorek, J. A. Swaffield, L. B. Jack, Fluid Mechanics, 5th Edition, 2005, ISBN-978-0-13-129293-2.

Thank You