RME 1102: Mechanical Engineering Fundamentals

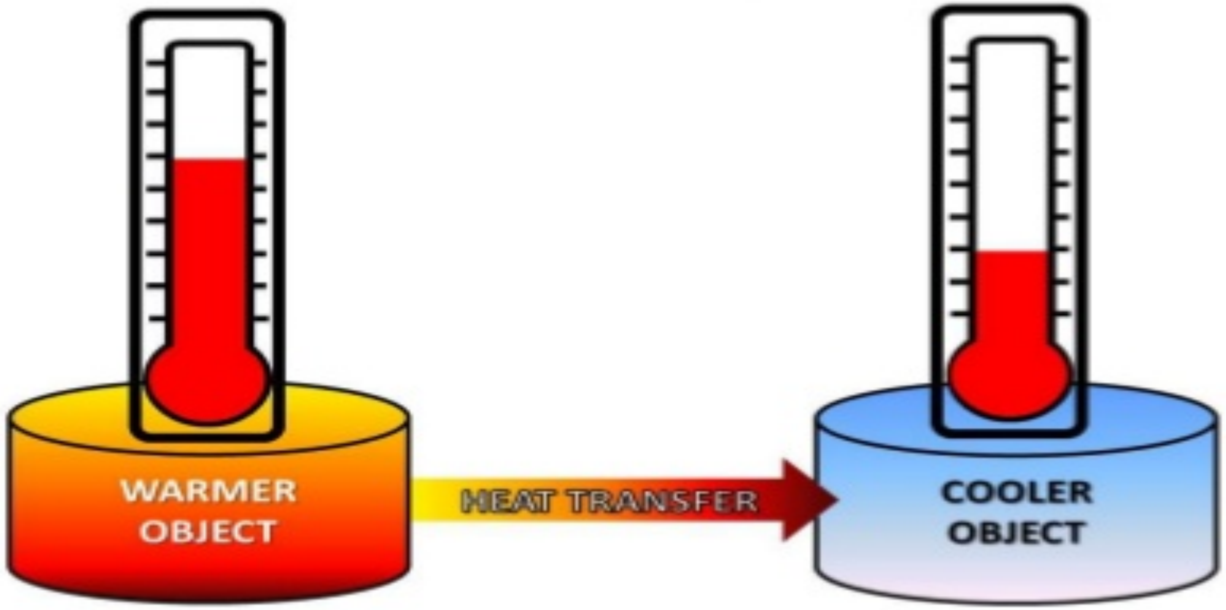
Dr. Abdul Aziz Shuvo
Assistant Professor
Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET), Dhaka-1000

Lecture 01

Fundamentals of Heat Transfer

Heat Transfer

Heat transfer is a study of the exchange of thermal energy through a body or between bodies which occurs when there is a temperature difference. When two bodies are at different temperatures, thermal energy **transfers** from the one with higher temperature to the one with lower temperature.

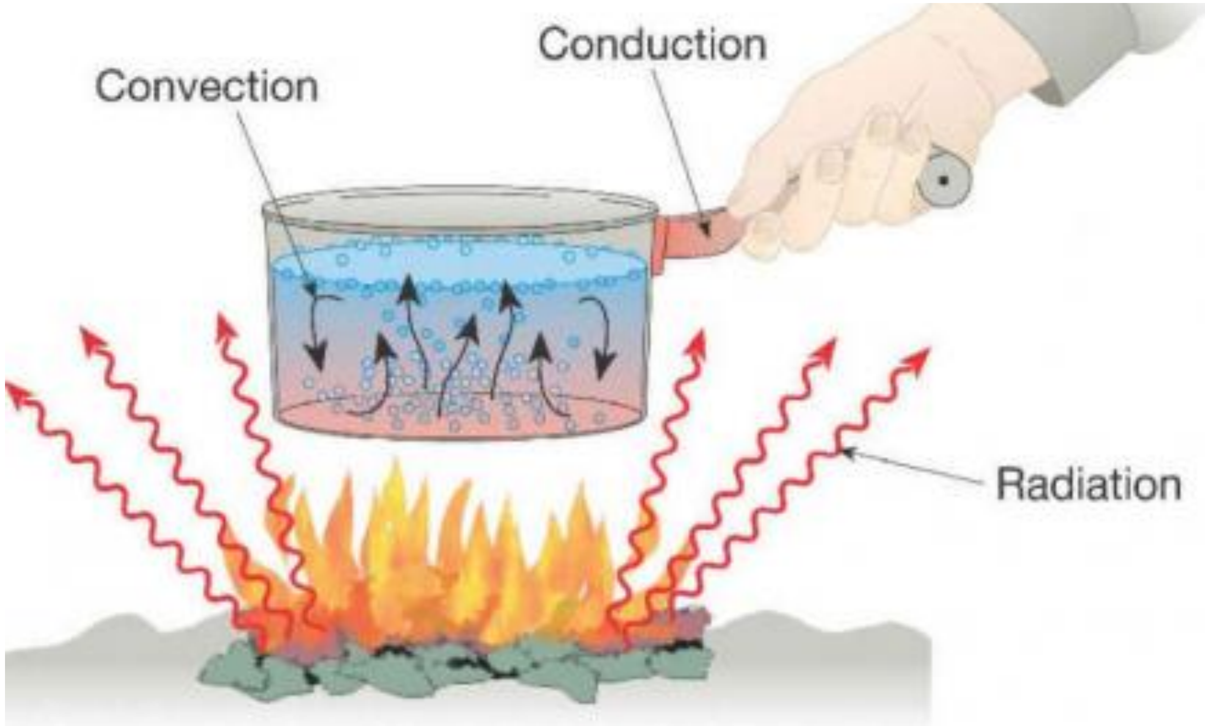


Basic Modes of Heat Transfer

Three modes of heat transfer:

a. Conduction
b. convection, and
c. radiation.

Any energy exchange between bodies occurs through one of these modes or a combination of them.

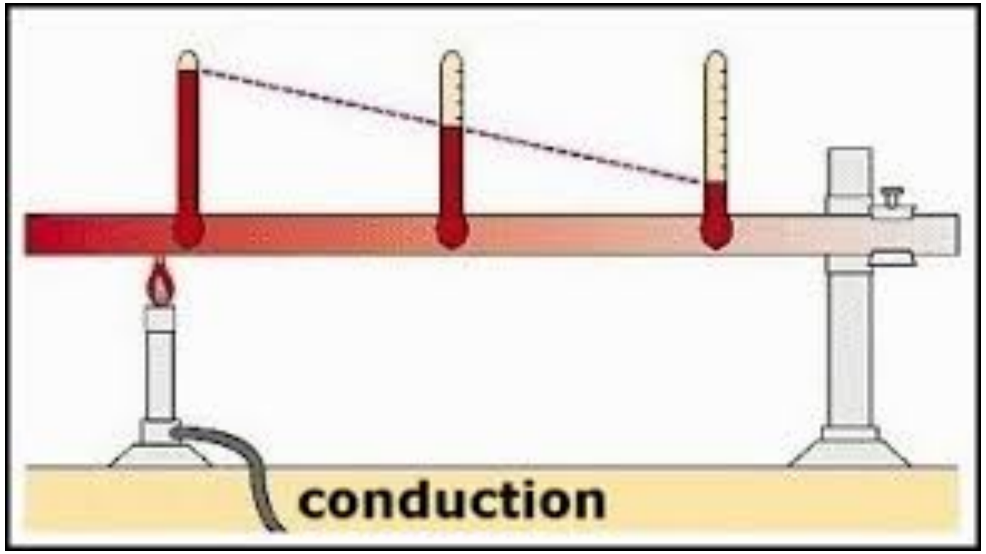


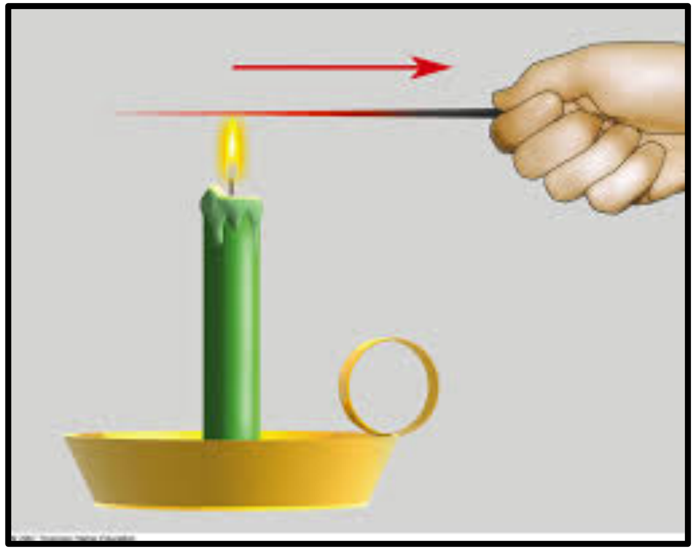
Conduction Heat Transfer

In **Conduction**, heat transfer takes place due to **temperature difference in a body** or **between bodies** in thermal contact, **without mixing** of mass. The rate of heat transfer through conduction is governed by the Fourier's law of heat conduction such as:

$$Q = -kA(dT/dx)$$

Where, 'Q' is the heat flow rate by conduction, 'k' is the thermal conductivity of body material, 'A' is the cross-sectional area normal to direction of heat flow, and 'dT/dx' is the temperature gradient of the section.



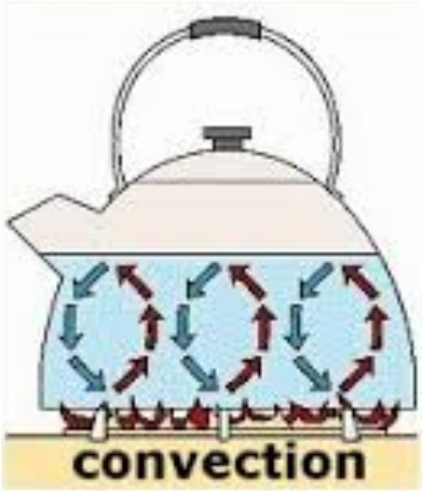


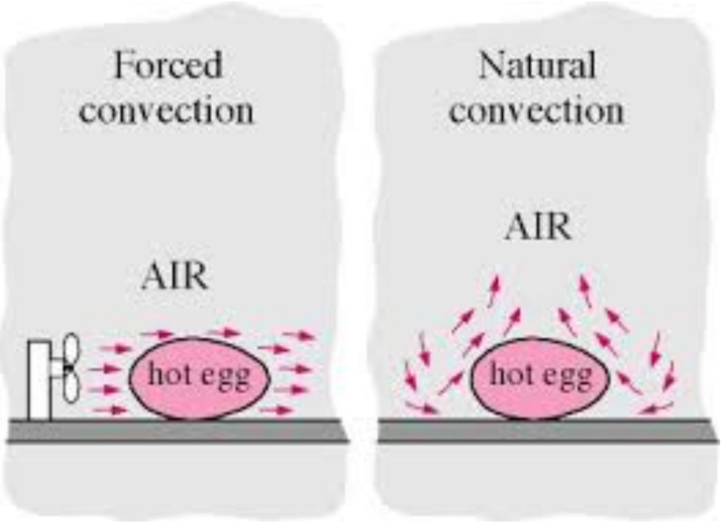
Convection Heat Transfer

In **convection**, heat is transferred to a moving fluid at the surface over which it flows by **combined molecular diffusion** and **bulk flow**. Convection involves conduction and fluid flow. The rate of convective heat transfer is governed by the *Newton's law of cooling.*

$$Q = hA(T_s - T_\infty)$$

Where 'T_s' is the surface temperature' 'T_∞' is the outside temperature 'h' is the coefficient of convection heat transfer.



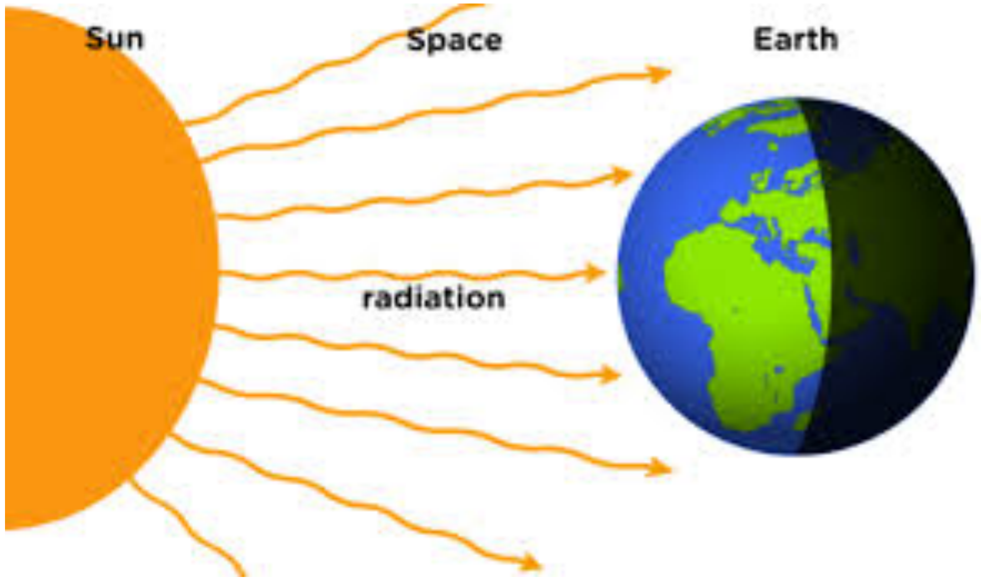


Radiation Heat Transfer

In **radiation**, heat is transferred in the form of **radiant energy as electromagnetic wave** from one body to another body. **No medium** for radiation to occur. The rate of heat radiation that can be emitted by a surface at a thermodynamic temperature is based on Stefan-Boltzmann law.

$$\mathbf{Q} = \sigma\varepsilon A s T_s^4$$

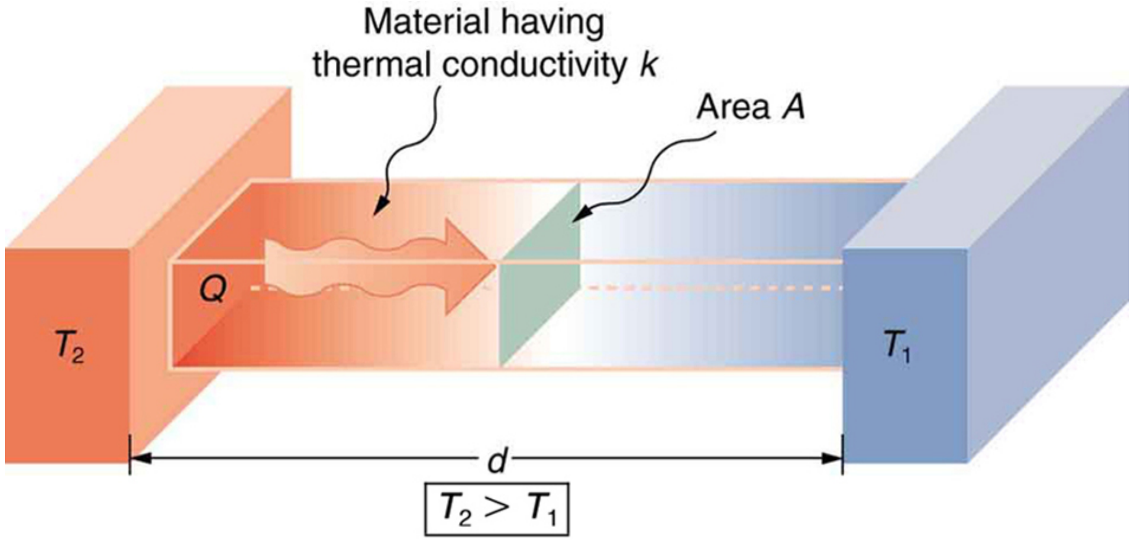
Where "σ" = 5.67 10 8W/m2·K is the Stefan–Boltzmann constant, "ε" is the surface emissivity, "T_s" is the absolute surface temperature, "A_s" is the surface area.



Thermal Conductivity

Recall the Fourier's law of heat conduction: **Q = -kA(dT/dx)**

Where, 'Q' is the heat flow rate by conduction (W), 'K' is the thermal conductivity of body material (W/m.K), 'A' is the cross-sectional area normal to direction of heat flow (m^2) and 'dT/dx' is the temperature gradient of the section (K/m).



k = -Q/A (dT/dx)

dT = T_2-T_1

dx = d

dT/dx = (T_2-T_1)/d

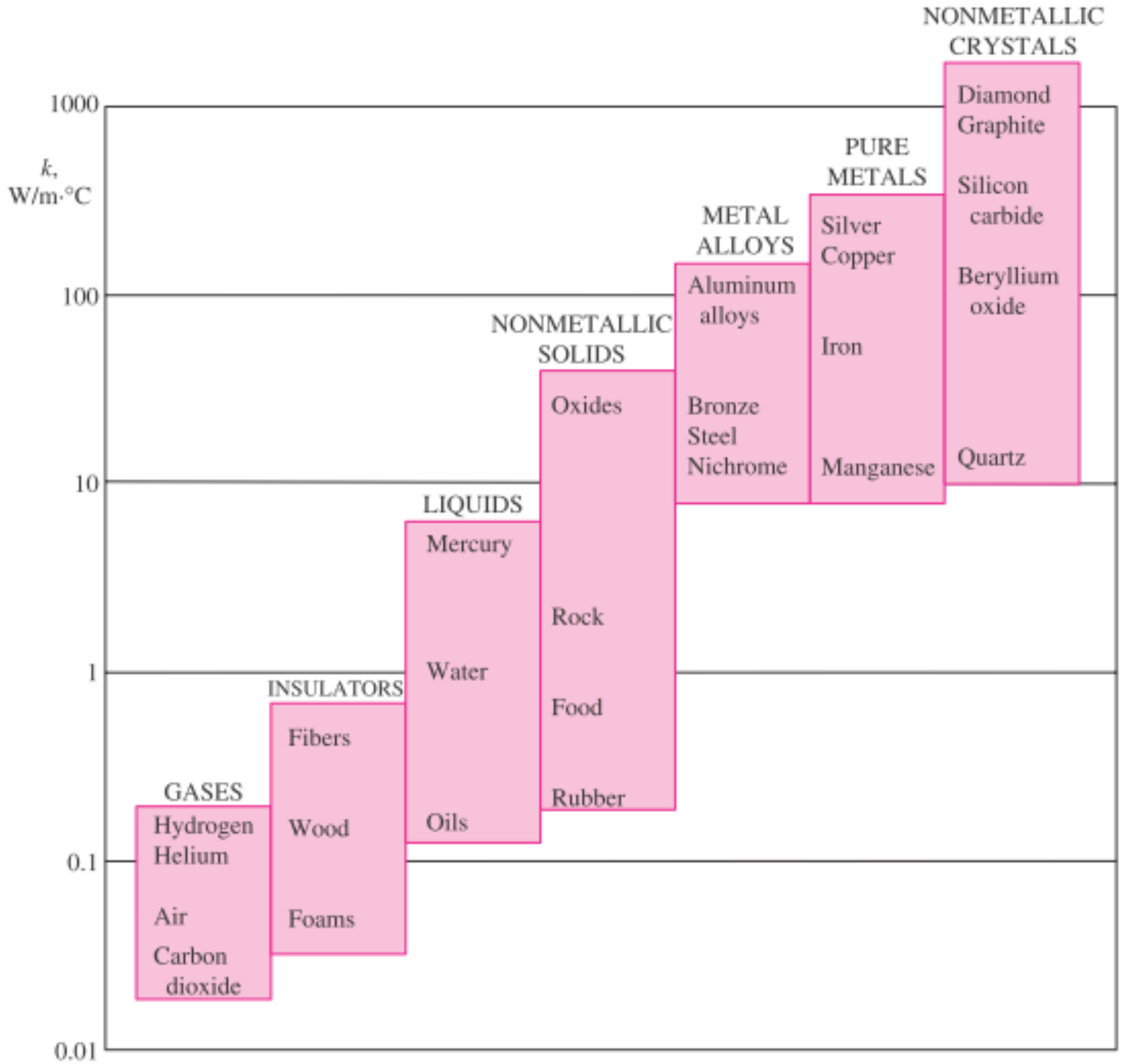
If A = 1 unit (m^2)

dT/dx = 1 unit (K/m), then

k = Q

Thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. A good thermal conductor has high thermal conductivity while material with low thermal conductivity is called insulator. Unit of thermal conductivity is W/m.K.

Thermal Conductivity of Various Materials

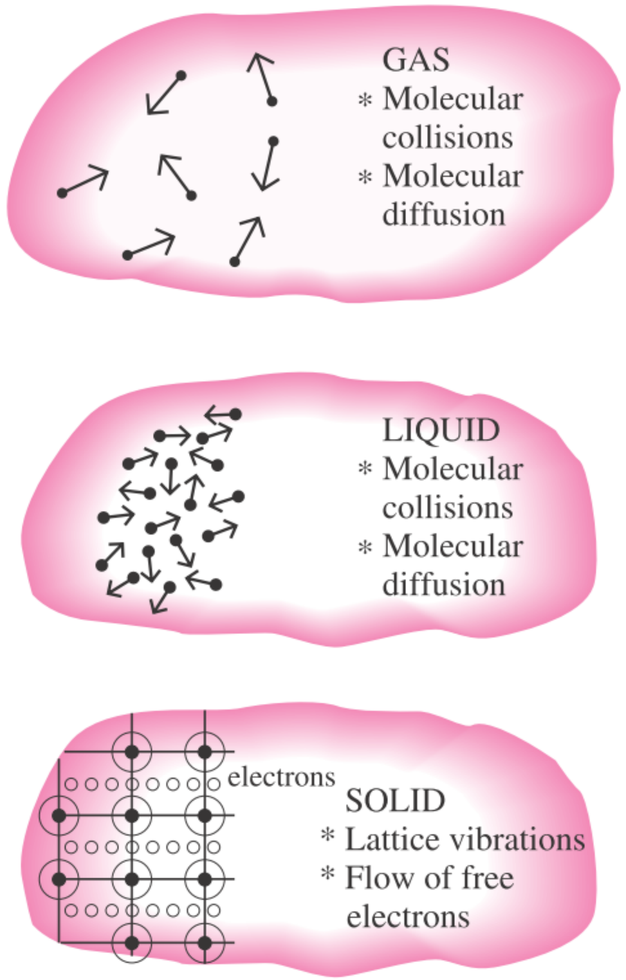


Thermal Conductivity of Gases, Liquids, and Solids

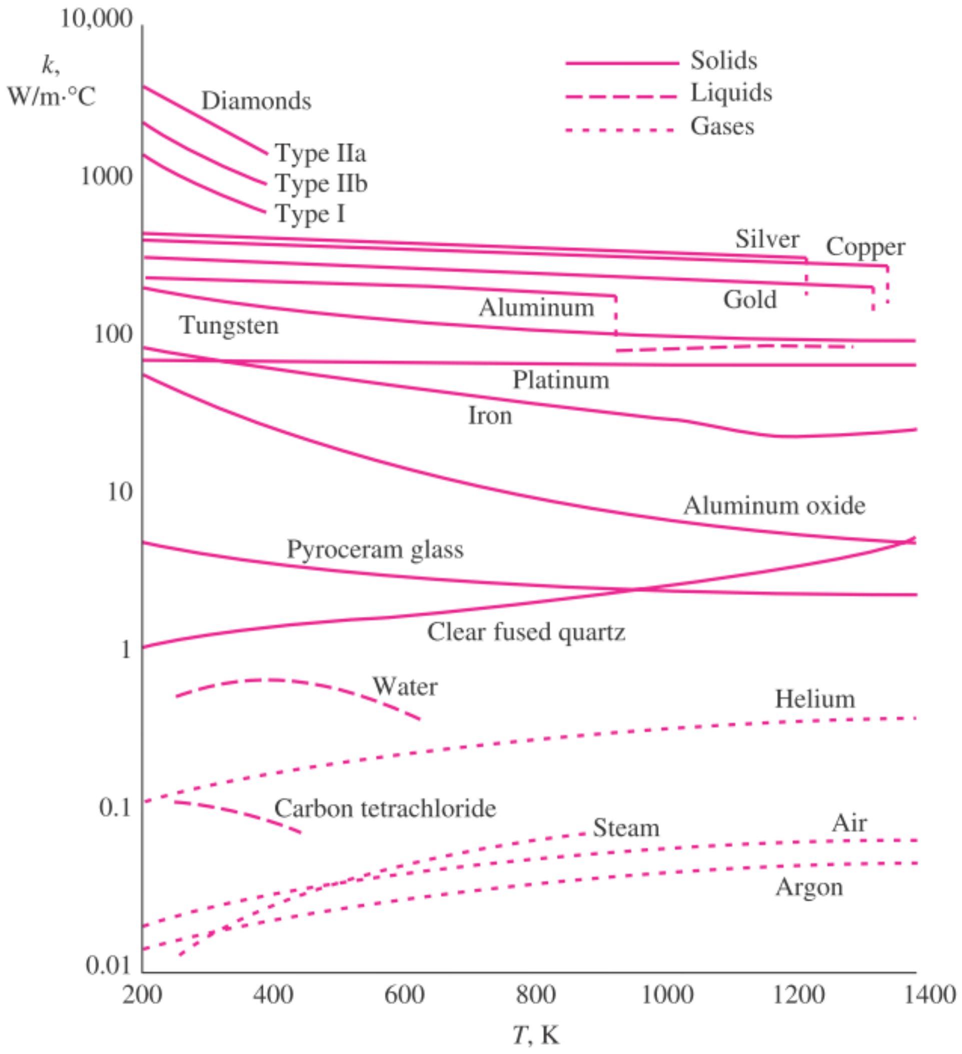
Thermal conductivity of **gases** is **proportional to the square root of the absolute temperature** T, and **inversely proportional to the square root of the molar mass M**.

Like gases, the **conductivity of liquids decreases with increasing molar mass**. Liquid metals such as **mercury and sodium have high thermal conductivities** and are very suitable for use in applications where a high heat transfer rate to a liquid is desired, as in **nuclear power plants**.

In solids, heat conduction is due to two effects: the **lattice vibrational waves** the **free flow of electrons** in the solid. The relatively high thermal conductivities of **pure metals** are primarily due to the **electronic component**. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is a highly ordered crystalline solid, has the highest known thermal conductivity at room temperature



Thermal Conductivity of Various Materials



Thermal conductivity of **gases gradually increases** with temperature

Unlike gases, the **conductivity of liquids decreases** with increasing temperature **except water** that firstly increases with temperature and then decreases.

In solids, the conductivity of solids **decreases with increasing temperature**. Some solids show tremendous raise in conductivity at very low temperature, for example: Cu

Thermal Diffusivity

The product ρC_p, (ρ being density (kg/m^3) and C_p is the specific heat (J/kg. °C)) is called the heat capacity of a material in unit volume basis (J/m^3.°C). Thermal diffusivity, which represents how fast heat diffuses through a material and is defined as:

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p} \qquad (\text{m}^2/\text{s})$$

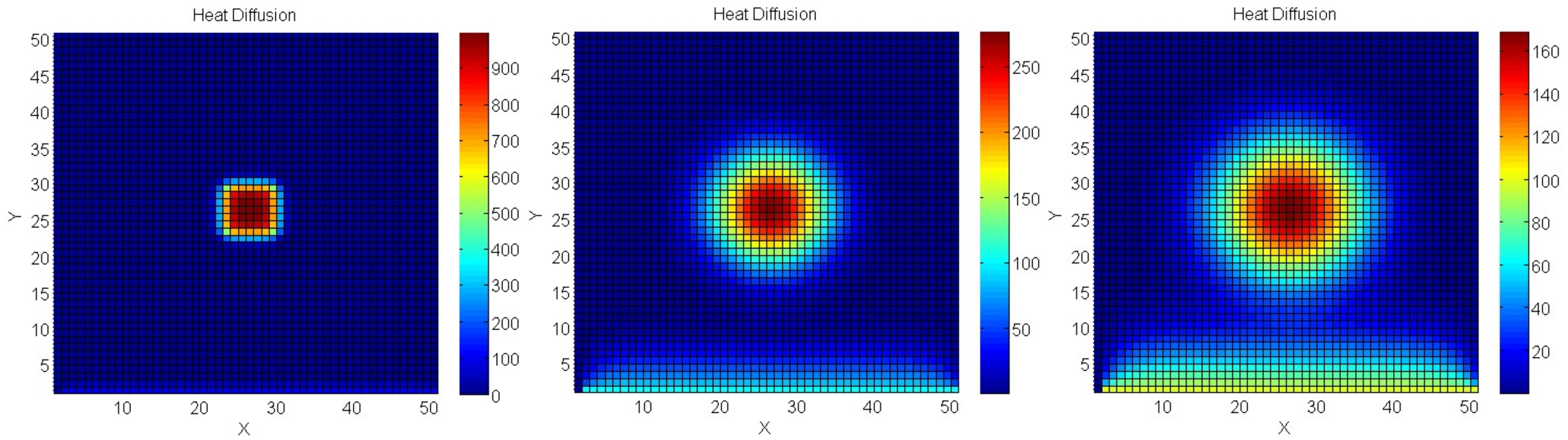
Thermal conductivity "k" represents **how well a material conducts heat**, and the **heat capacity**, "ρC_p" represents **how much energy a material stores** per unit volume. The thermal diffusivity of a material can be viewed as the ratio of the heat conducted through the material to the heat stored per unit volume.

A material that has a **high thermal conductivity** or a **low heat capacity** will obviously have a large thermal diffusivity.

The **larger** the thermal diffusivity, the **faster** the propagation of heat into the medium.

A **small** value of thermal diffusivity means that heat is **mostly absorbed** by the material and a **small amount** of heat will be **conducted** further

Thermal Diffusivity



Let us consider **sudden release of heat** from a point source.

Heat will propagate in **all directions from a point** just after been released. It is similar **to growing up of a sphere**. With time heat will propagate forming an imaginary sphere **getting bigger and bigger**.

The **propagation rate** is related to the **rate of increase of the sphere surface** area and hence any **propagation parameter** in 3D has the unit of **m^2/s**.

Thermodynamics vs Heat Transfer

Thermodynamics	Heat Transfer
Concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another.	Deals with the determination of the rates of such energy transfers is the heat transfer
Deals with the amount of energy in form of heat or work during a process and only considers the start and end states in equilibrium. No tension about time	Deals with the rate of energy transfer thus, it gives idea of how long a heat transfer will occur.
Deals with equilibrium phenomena	Deals with time and non equilibrium phenomena. Heat can only transfer when there is a temperature gradient exists in a body and which is indication of non equilibrium phenomena

In short, Thermodynamics gives "**Why**" a process will occur and Heat Transfer will tell, "**How**" a process will occur when there is a concern about **transfer of heat**.

Classical Heat Transfer and Continuum Hypothesis

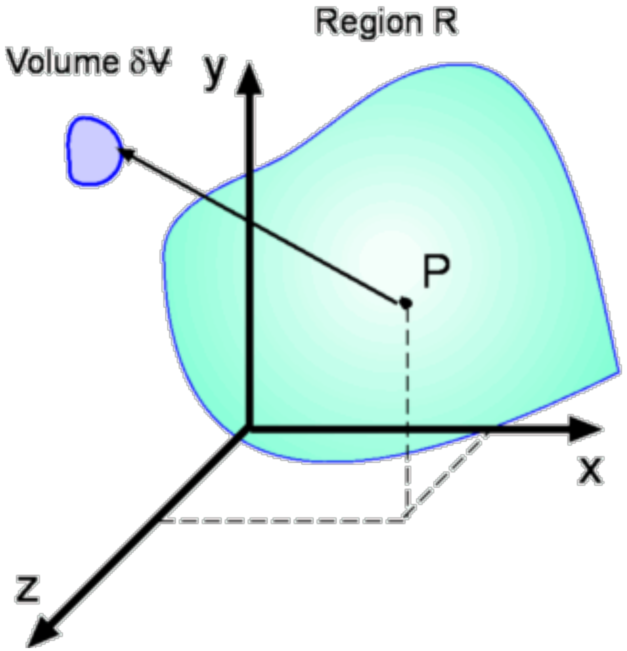
Application of classical heat transfer equation requires the **adoption of continuum approach**. That is the medium is assumed to be continuous and indefinitely divisible. To judge the continuity of medium one approach is to consider the **Knudsen Number (Kn)**:

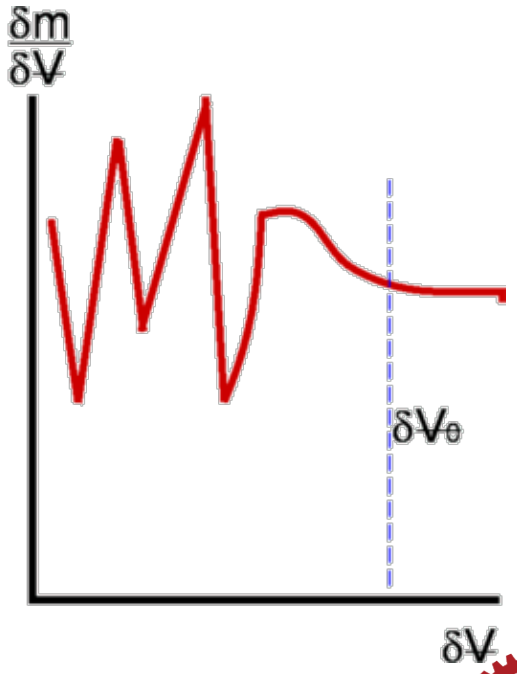
Kn = Mean free path (λ)/Characteristic Length (L_c)

Mean Free Path (λ): Distance traveled by an atom between two successive collision

Characteristic Length (L_c): System Size Parameter (Length/Width/Height which one is minimum)

Kn << 0.1 for the application of Fourier's Law.

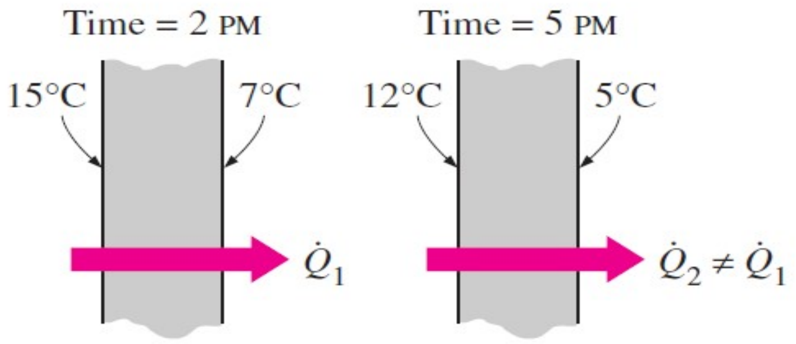




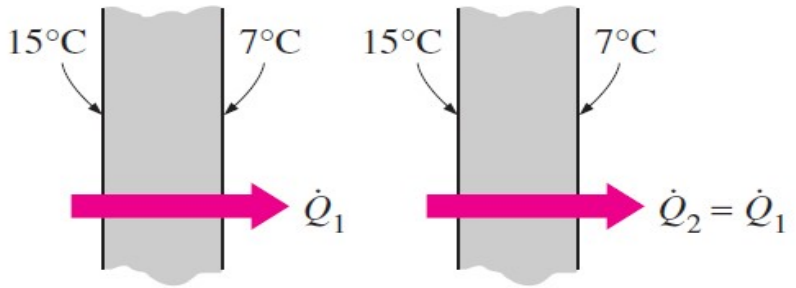
Steady vs Transient Heat Transfer

- **Heat transfer problems are often classified as being steady (also called steady state) or transient (also called unsteady).**

- **The term steady implies no change with time at any point within the medium, while transient implies variation with time or time dependence.**



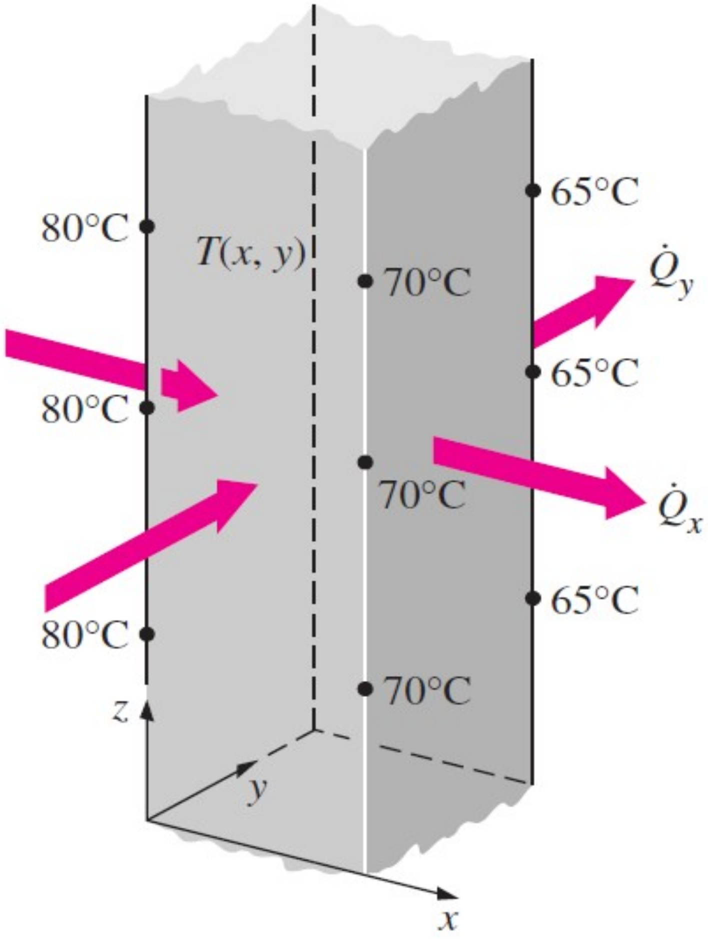
(*a*) Transient



(*b*) Steady-state

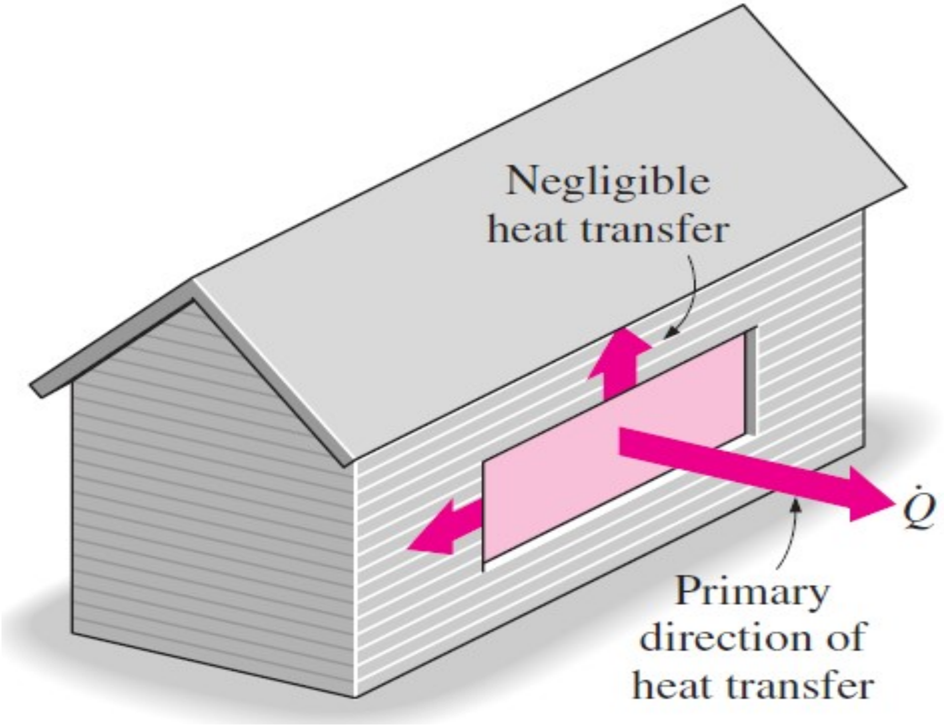
- Heat transfer problems are also classified as being one-dimensional, two-dimensional, or three-dimensional, depending on the relative magnitudes of heat transfer rates in different directions and the level of accuracy desired.

- The temperature in a medium, in some cases, varies mainly in two primary directions, and the variation of temperature in the third direction (and thus heat transfer in that direction) is negligible. A heat transfer problem in that case is said to be two-dimensional.



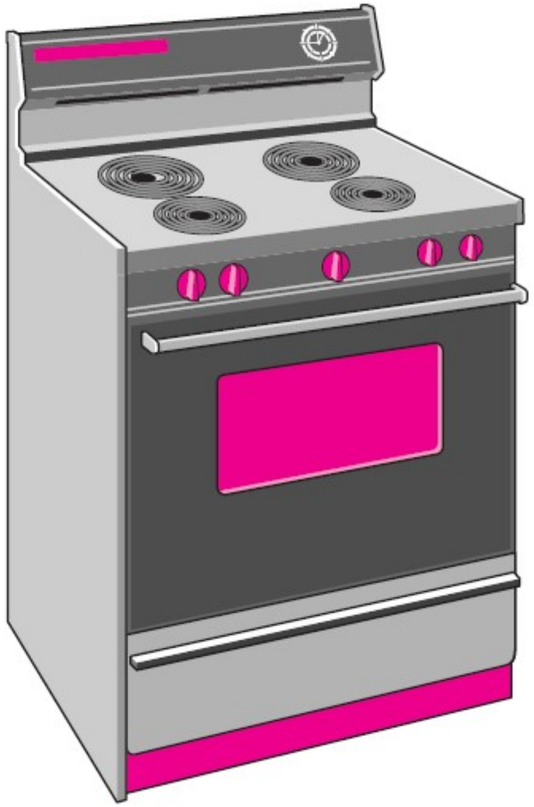
Multidimensional Heat Transfer

- **A heat transfer problem is said to be one-dimensional if the temperature in the medium varies in one direction only and thus heat is transferred in one direction, and the variation of temperature and thus heat transfer in other directions are negligible or zero.**



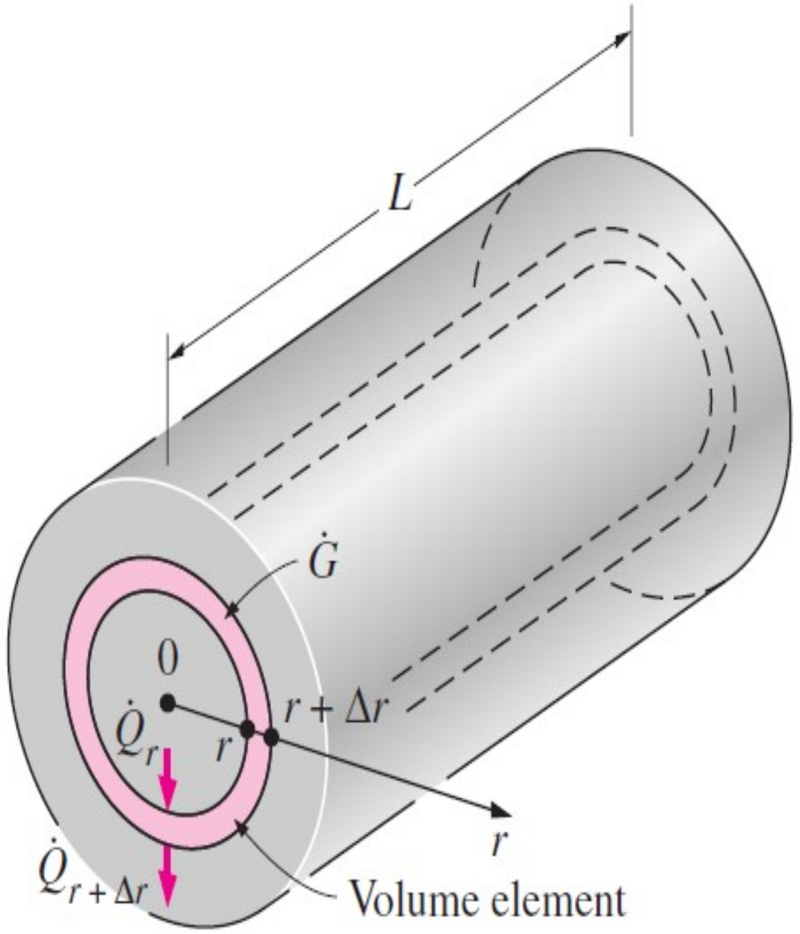
Heat Generation

- A medium through which heat is conducted may involve the conversion of electrical, nuclear, or chemical energy into heat (or thermal) energy. In heat conduction analysis, such conversion processes are characterized as heat generation.



Heat Conduction in a Long Cylinder

Consider a thin cylindrical shell element of thickness Δr in a long cylinder, as shown in figure. Assume the density of the cylinder is ρ , the specific heat is C, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location.



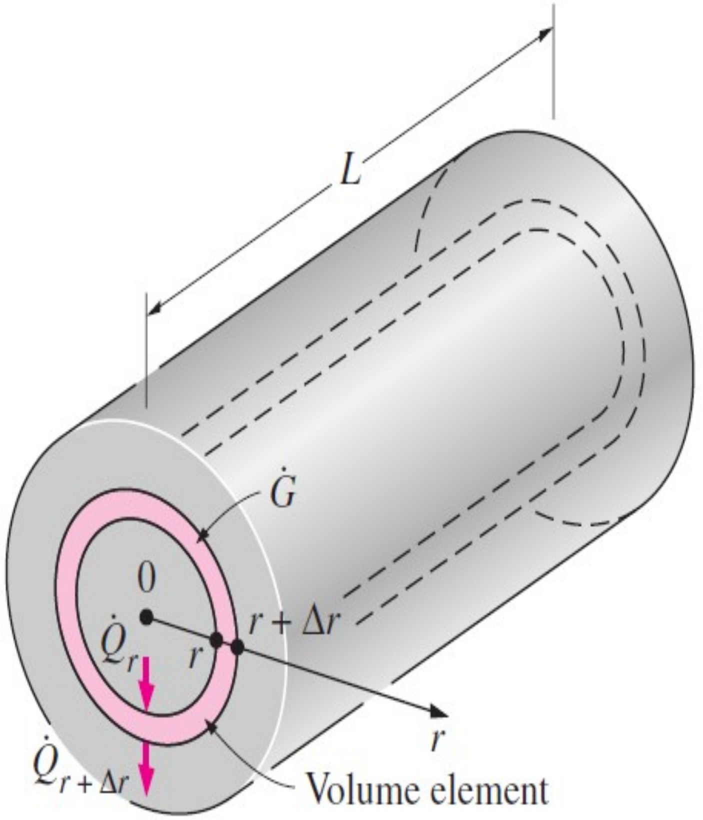
Heat Conduction in a Long Cylinder

An energy balance on this thin cylindrical shell element during a small time interval t can be expressed as:

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r+\Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$



Heat Conduction in a Long Cylinder

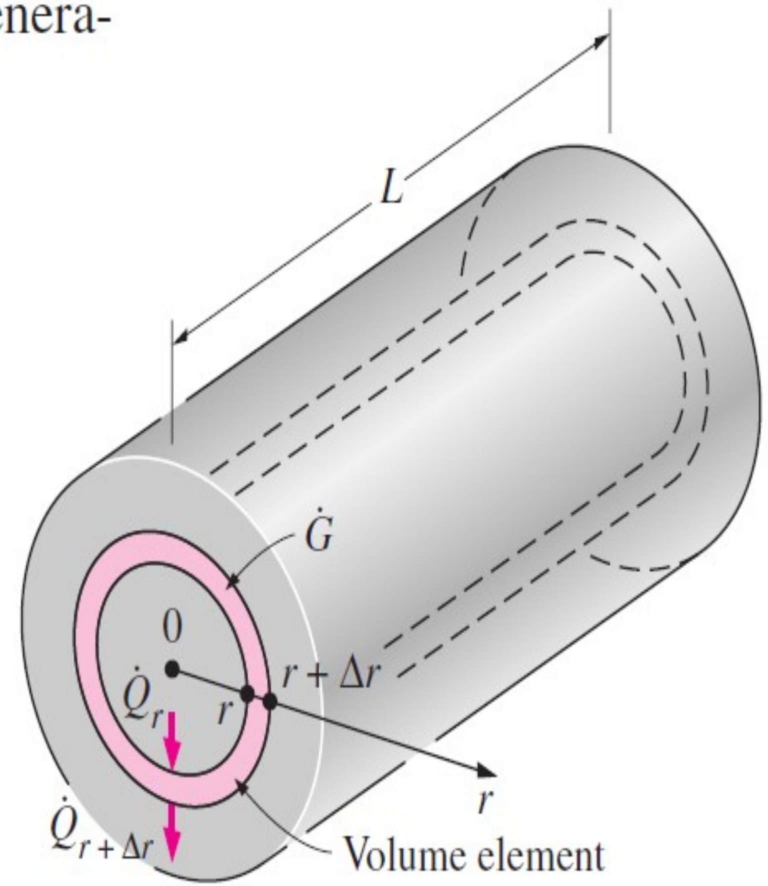
The change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta r(T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}A\Delta r$$

Substituting into energy balance equation, we get

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{g}A\Delta r = \rho CA\Delta r\frac{T_{t+\Delta t} - T_t}{\Delta t}$$



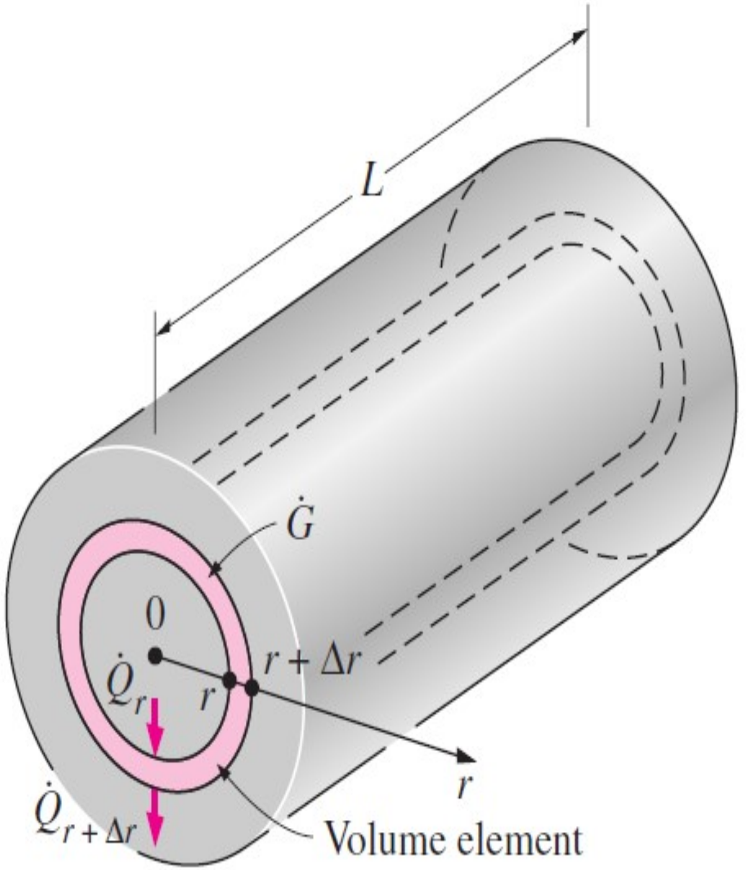
Heat Conduction in a Long Cylinder

where $A = 2\pi rL$. You may be tempted to express the area at the *middle* of the element using the *average* radius as $A = 2\pi(r + \Delta r/2)L$. But there is nothing we can gain from this complication since later in the analysis we will take the limit as $\Delta r \rightarrow 0$ and thus the term $\Delta r/2$ will drop out. Now dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \rightarrow 0$ and $\Delta t \rightarrow 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$



Heat Conduction in a Long Cylinder

since, from the definition of the derivative and Fourier's law of heat conduction,

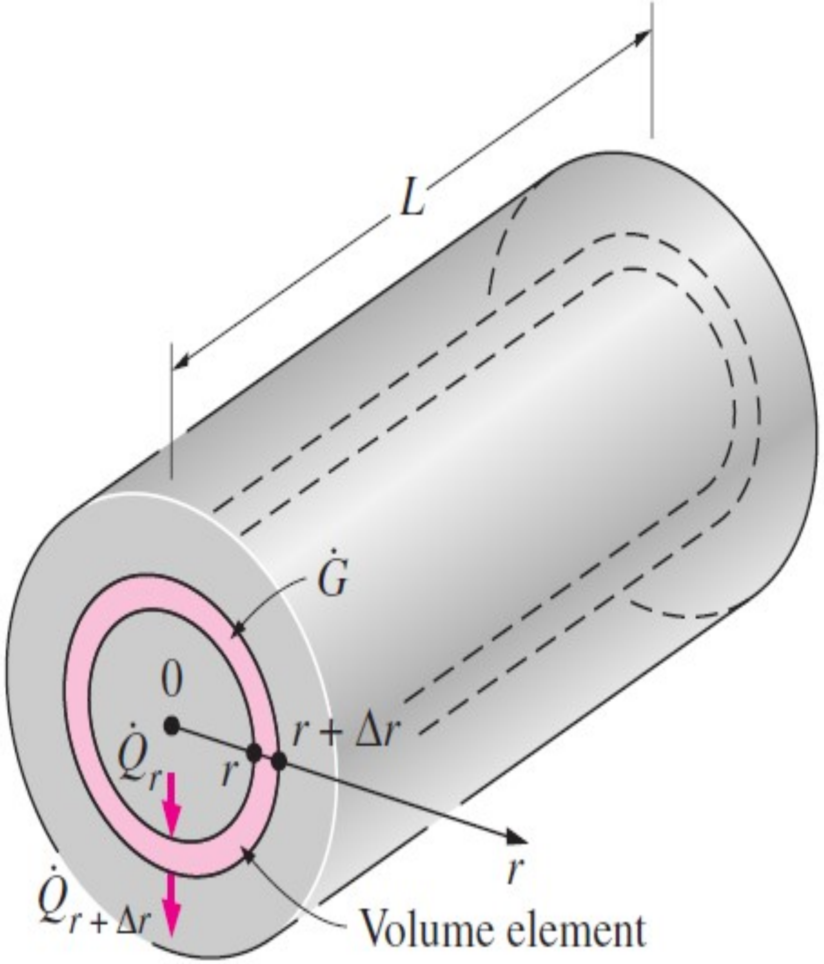
$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 2\pi rL$, the one-dimensional transient heat conduction equation in a cylinder becomes

Variable conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

For the case of constant thermal conductivity, the equation above reduces to

Constant conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



Heat Conduction in a Long Cylinder

(1) *Steady-state:* $(\partial/\partial t = 0)$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0 \text{ and } \dot{g} = 0)$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Thermal Boundary Conditions (BC's)

- Appropriate Boundary Condition and Initial Condition are Necessary for the solution of Heat Conduction equation.

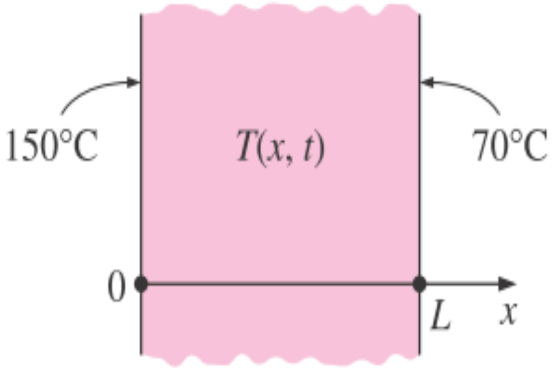
- Order of differentiation with respect to any dimension/direction is two while with respect to time it is one. So at least two Boundary Conditions are necessary for each space dimension/direction while only one condition with respect to any instant of time must be known.

- Boundary Conditions (BC) are the mathematical expression of thermal condition at the boundary of the system.

- Initial Condition (IC) is the mathematical expression of thermal condition of the system at the instant $t = 0$

Thermal Boundary Conditions (BC's)

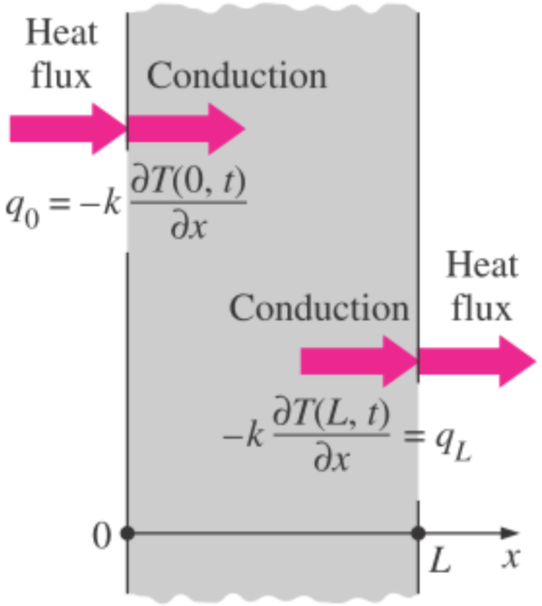
1. Specified Temperature Boundary Condition



$T(0, t) = 150°C$

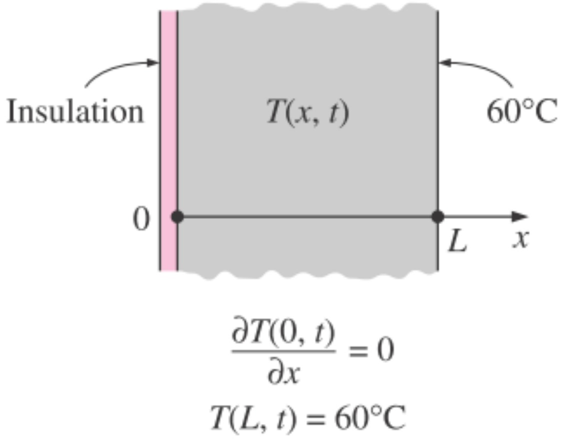
$T(L, t) = 70°C$

2. Specified Heat Flux Boundary Condition: Heat flux, q = Heat Transfer rate per unit area (W/m²)

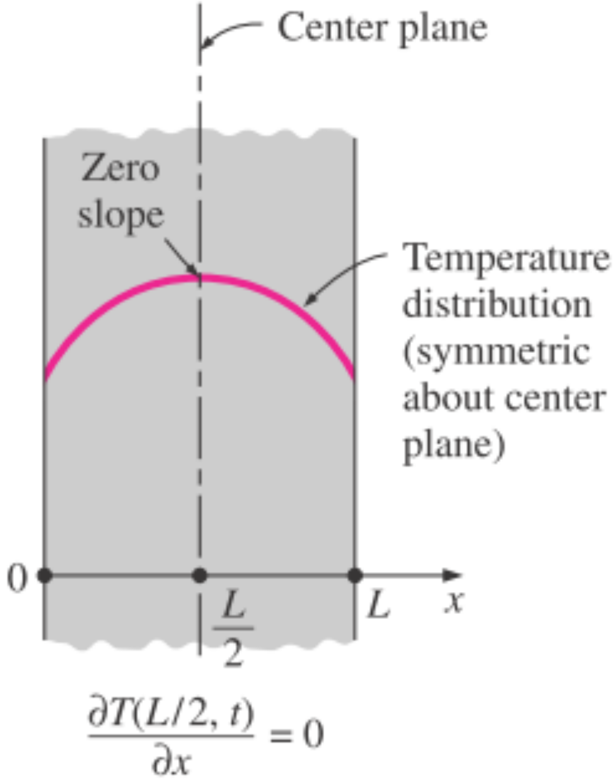


Thermal Boundary Conditions (BC's)

2. A. Insulated Boundary Condition: (Boundary Heat Flux, q = 0)

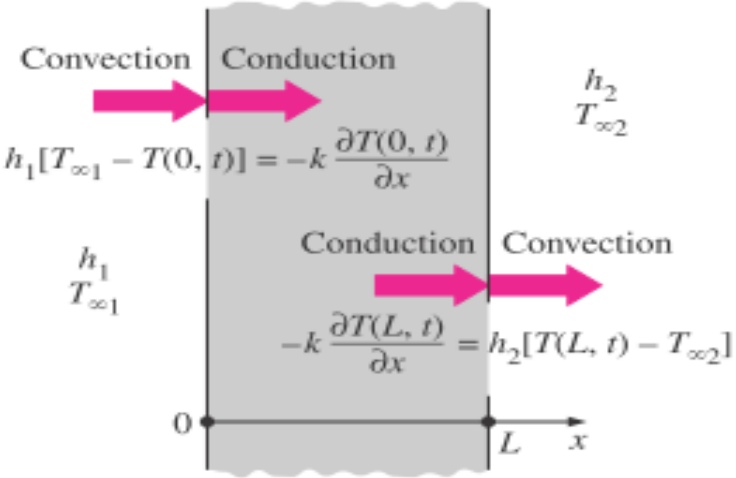


2.B. Symmetry Boundary Condition:
(Any Flux at the line of symmetry = 0)



Thermal Boundary Conditions (BC's)

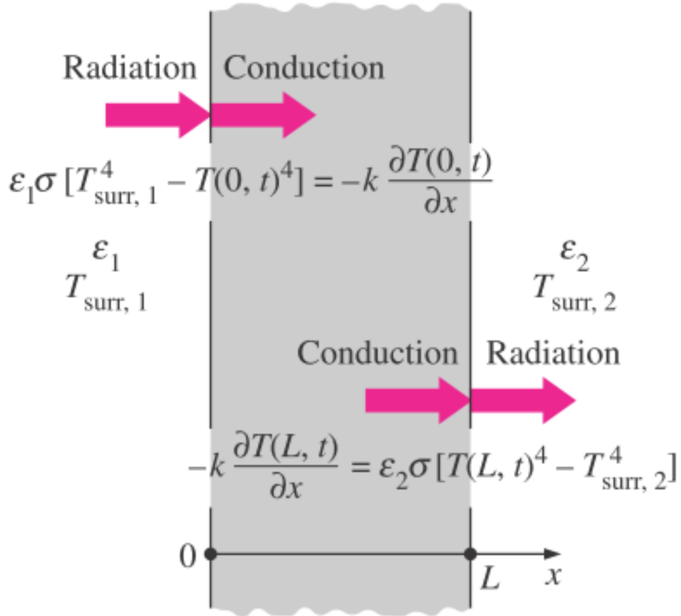
3. Convective Boundary Condition



$$\begin{pmatrix}\text{Heat conduction} \\ \text{at the surface in a} \\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Heat convection} \\ \text{at the surface in} \\ \text{the same direction}\end{pmatrix}$$

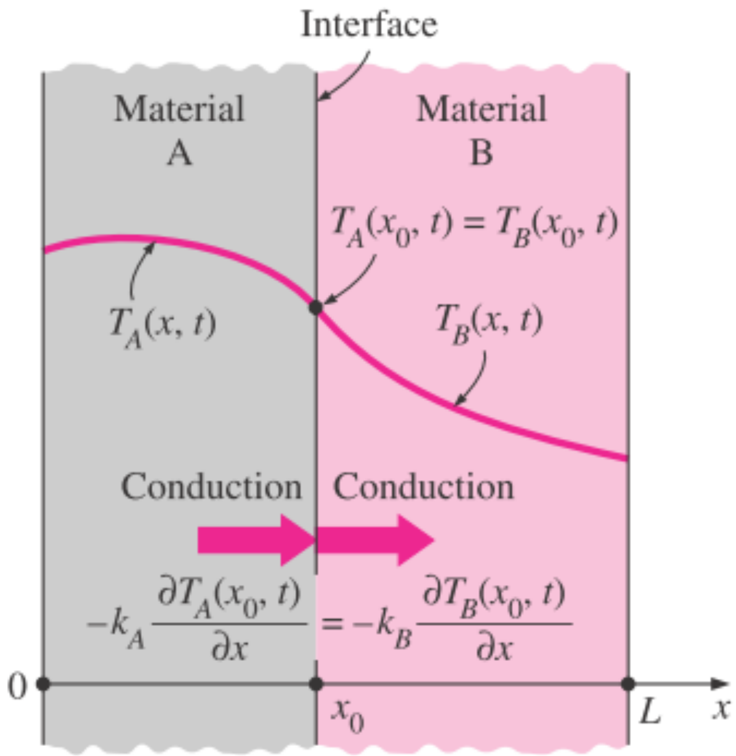
4. Radiation Boundary Condition

$$\begin{pmatrix}\text{Heat conduction} \\ \text{at the surface in a} \\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Radiation exchange} \\ \text{at the surface in} \\ \text{the same direction}\end{pmatrix}$$



Thermal Boundary Conditions (BC's)

5. Interface Boundary Condition



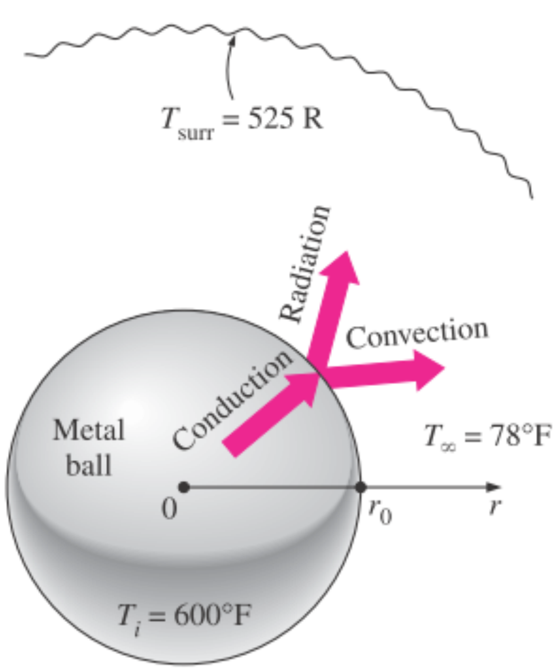
The boundary conditions at an interface are based on the requirements that:

(1) two bodies in contact must have **the same temperature at the area of contact** and

(2) an interface (which is a surface) cannot store any energy, and thus the **heat flux on the two sides of an interface must be the same**.

Thermal Boundary Conditions (BC’s)

6. Generalized Boundary Condition



$$\left(\begin{array}{c}\text{Heat transfer}\\ \text{to the surface}\\ \text{in all modes}\end{array}\right) = \left(\begin{array}{c}\text{Heat transfer}\\ \text{from the surface}\\ \text{in all modes}\end{array}\right)$$

- So far **single mode heat transfer**, such as the specified heat flux, convection, or radiation have been considered for simplicity.
- However, a surface may involve **multimodal heat transfer** such as convection, radiation, and specified heat flux simultaneously.
- The boundary condition in such cases is again obtained from a **surface energy balance**, expressed as:

EXAMPLE 2–15

Consider a steam pipe of length $L = 20$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 8$ cm, and thermal conductivity $k = 20$ W/m · °C, as shown in Figure 2–50. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 150°C$ and $T_2 = 60°C$, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

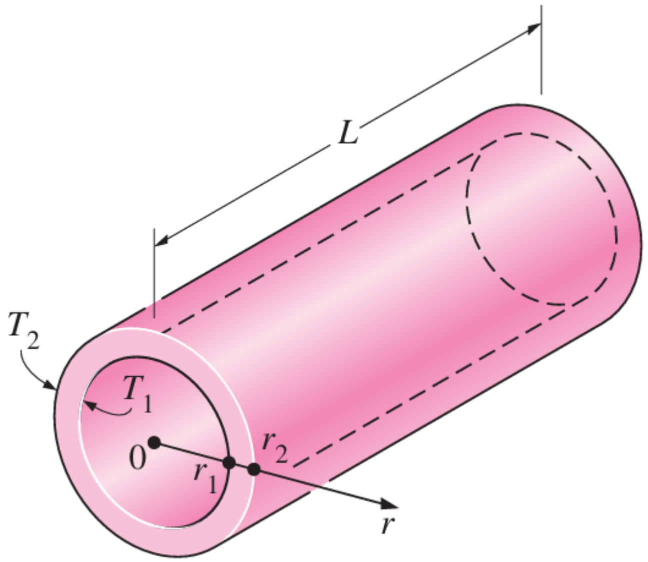


FIGURE 2–50
Schematic for Example 2–15.

EXAMPLE 2–15

SOLUTION A steam pipe is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 20$ W/m · °C.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 150°\text{C}$$
$$T(r_2) = T_2 = 60°\text{C}$$

Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Again integrating with respect to r gives (Fig. 2–51)

$$T(r) = C_1 \ln r + C_2 \qquad \textbf{(a)}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (*a*) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$
$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \qquad \text{and} \qquad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)}\ln r_1$$

Substituting them into Eq. (*a*) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\right)(T_2 - T_1) + T_1 \qquad \textbf{(2-58)}$$

The rate of heat loss from the steam is simply the total rate of heat conduction through the pipe, and is determined from Fourier's law to be

EXAMPLE 2–15

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)} \quad \textbf{(2-59)}$$

The numerical value of the rate of heat conduction through the pipe is determined by substituting the given values

$$\dot{Q} = 2\pi(20\ \text{W/m}\cdot{}^\circ\text{C})(20\ \text{m})\frac{(150 - 60)^\circ\text{C}}{\ln(0.08/0.06)} = \mathbf{786\ kW}$$

DISCUSSION Note that the total rate of heat transfer through a pipe is constant, but the heat flux is not since it decreases in the direction of heat transfer with increasing radius since $\dot{q} = \dot{Q}/(2\pi rL)$.

EXAMPLE 2–19

Consider a long resistance wire of radius $r_1 = 0.2$ cm and thermal conductivity $k_{wire} = 15$ W/m · °C in which heat is generated uniformly as a result of resistance heating at a constant rate of $\dot{g} = 50$ W/cm^3 (Fig. 2–61). The wire is embedded in a 0.5-cm-thick layer of ceramic whose thermal conductivity is $k_{ceramic} = 1.2$ W/m · °C. If the outer surface temperature of the ceramic layer is measured to be $T_s = 45$°C, determine the temperatures at the center of the resistance wire and the interface of the wire and the ceramic layer under steady conditions.

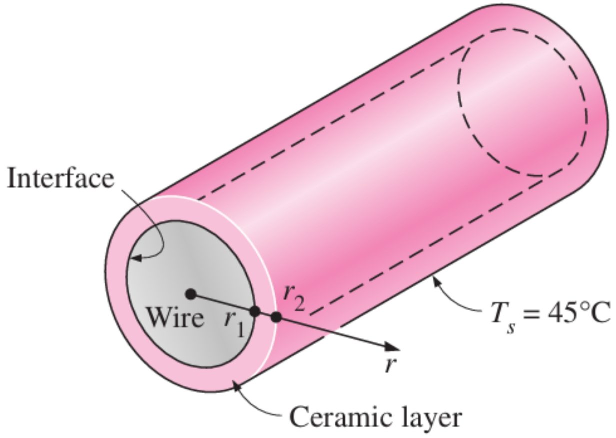


FIGURE 2–61
Schematic for Example 2–19.

EXAMPLE 2–19

SOLUTION The surface and interface temperatures of a resistance wire covered with a ceramic layer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since this two-layer heat transfer problem possesses symmetry about the centerline and involves no change in the axial direction, and thus $T = T(r)$. **3** Thermal conductivities are constant. **4** Heat generation in the wire is uniform.

Properties It is given that $k_{\text{wire}} = 15$ W/m · °C and $k_{\text{ceramic}} = 1.2$ W/m · ° C.

Analysis Letting T_I denote the unknown interface temperature, the heat transfer problem in the wire can be formulated as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right) + \frac{\dot{g}}{k} = 0$$

with

$$T_{\text{wire}}(r_1) = T_I$$
$$\frac{dT_{\text{wire}}(0)}{dr} = 0$$

This problem was solved in Example 2–18, and its solution was determined to be

$$T_{\text{wire}}(r) = T_I + \frac{\dot{g}}{4k_{\text{wire}}}(r_1^2 - r^2) \qquad \textbf{(a)}$$

Noting that the ceramic layer does not involve any heat generation and its outer surface temperature is specified, the heat conduction problem in that layer can be expressed as

$$\frac{d}{dr}\left(r\frac{dT_{\text{ceramic}}}{dr}\right) = 0$$

with

$$T_{\text{ceramic}}(r_1) = T_I$$
$$T_{\text{ceramic}}(r_2) = T_s = 45°\text{C}$$

This problem was solved in Example 2–15, and its solution was determined to be

$$T_{\text{ceramic}}(r) = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_s - T_I) + T_I \qquad \textbf{(b)}$$

We have already utilized the first interface condition by setting the wire and ceramic layer temperatures equal to T_I at the interface $r = r_1$. The interface temperature T_I is determined from the second interface condition that the heat flux in the wire and the ceramic layer at $r = r_1$ must be the same:

$$-k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr} = -k_{\text{ceramic}}\frac{dT_{\text{ceramic}}(r_1)}{dr} \quad \rightarrow \quad \frac{\dot{g}r_1}{2} = -k_{\text{ceramic}}\frac{T_s - T_I}{\ln(r_2/r_1)}\left(\frac{1}{r_1}\right)$$

Solving for T_I and substituting the given values, the interface temperature is determined to be

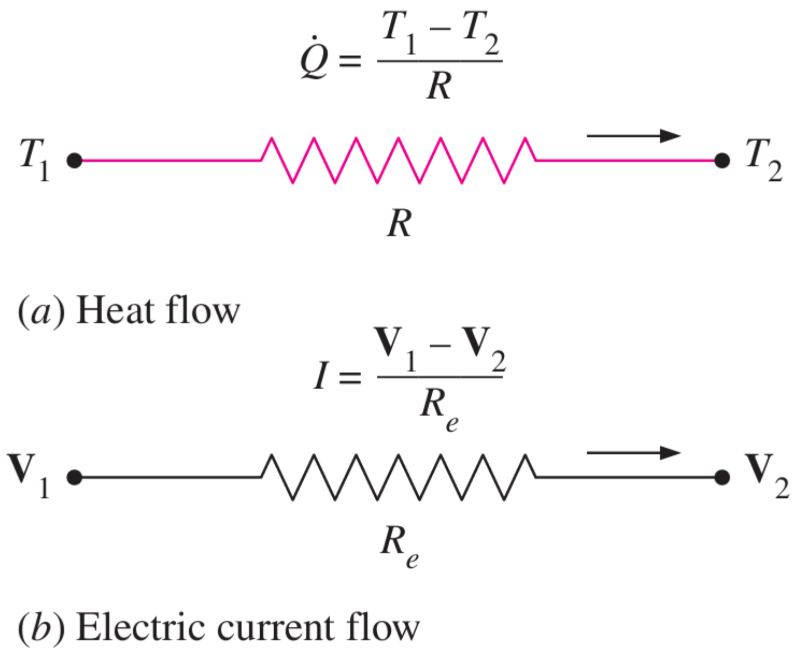
$$T_I = \frac{\dot{g}r_1^2}{2k_{\text{ceramic}}}\ln\frac{r_2}{r_1} + T_s$$
$$= \frac{(50 \times 10^6 \text{ W/m}^3)(0.002 \text{ m})^2}{2(1.2 \text{ W/m} \cdot °\text{C})}\ln\frac{0.007 \text{ m}}{0.002 \text{ m}} + 45°\text{ C} = \mathbf{149.4°C}$$

Knowing the interface temperature, the temperature at the centerline ($r = 0$) is obtained by substituting the known quantities into Eq. (*a*),

$$T_{\text{wire}}(0) = T_I + \frac{\dot{g}r_1^2}{4k_{\text{wire}}} = 149.4°\text{C} + \frac{(50 \times 10^6 \text{ W/m}^3)(0.002 \text{ m})^2}{4 \times (15 \text{ W/m} \cdot °\text{C})} = \mathbf{152.7°C}$$

In the present analysis following assumption are to be made

- ➢1-D, Steady Heat Conduction
- ➢No Heat Generation
- ➢Constant Property



- ❖ *Rate of heat transfer* through a layer corresponds to the electric current.
- ❖ *Temperature difference* corresponds to *voltage difference* across the layer.
- ❖ *Thermal resistance* corresponds to *electrical resistance*.

Thermal Resistance Concept

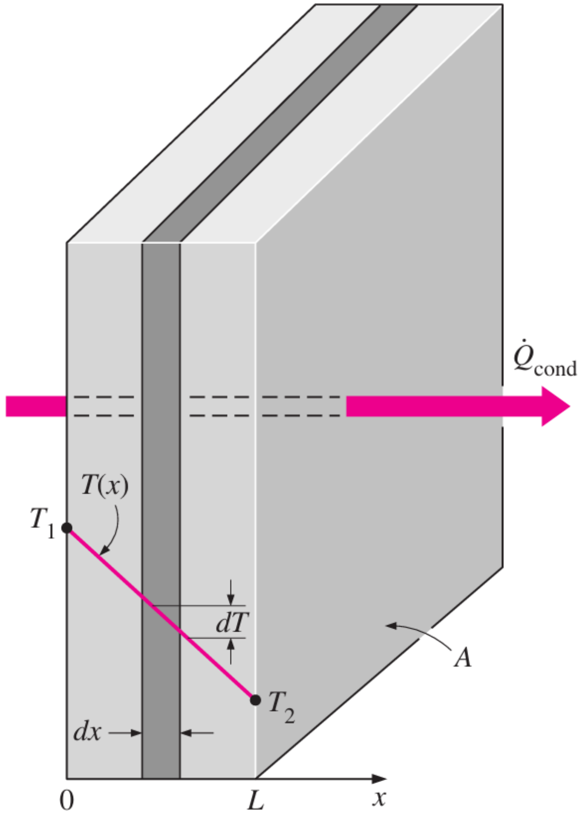
$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}}$$

$$R_{\text{wall}} = \frac{L}{kA}$$



$\boldsymbol{R_{\text{cond}}}$ is the thermal resistance of the wall against heat conduction or simply the conduction resistance of the wall. Note that the thermal resistance of a medium depends on the geometry and the thermal properties of the medium.

Thermal Resistance Concept

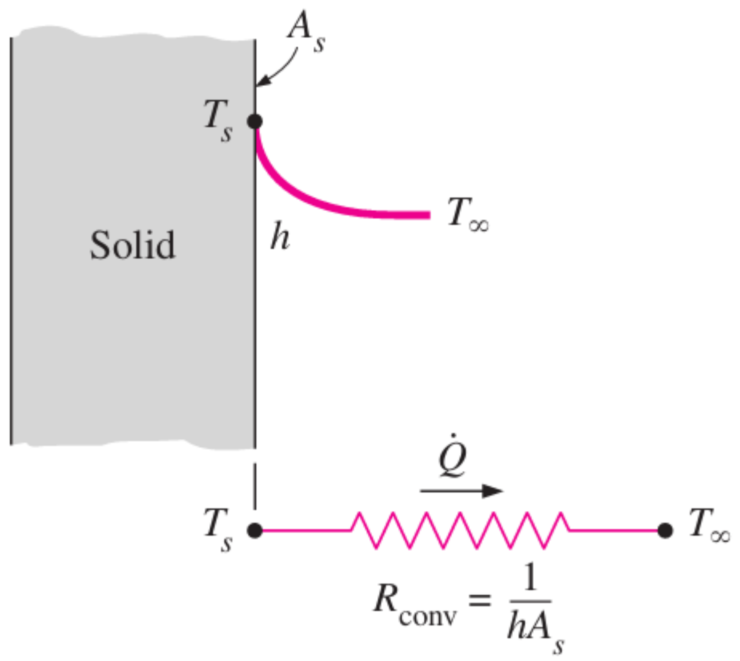
Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is $T\infty$, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate can be rearranged as:

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}}$$

where,

$$R_{\text{conv}} = \frac{1}{hA_s} \qquad (°\text{C/W})$$



R**conv** is the thermal resistance of the surface against heat convection, or simply the **convection resistance** of the surface.

Thermal Resistance Concept

When the wall is surrounded by a gas, the radiation effects, which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s (T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \qquad \text{(W)}$$

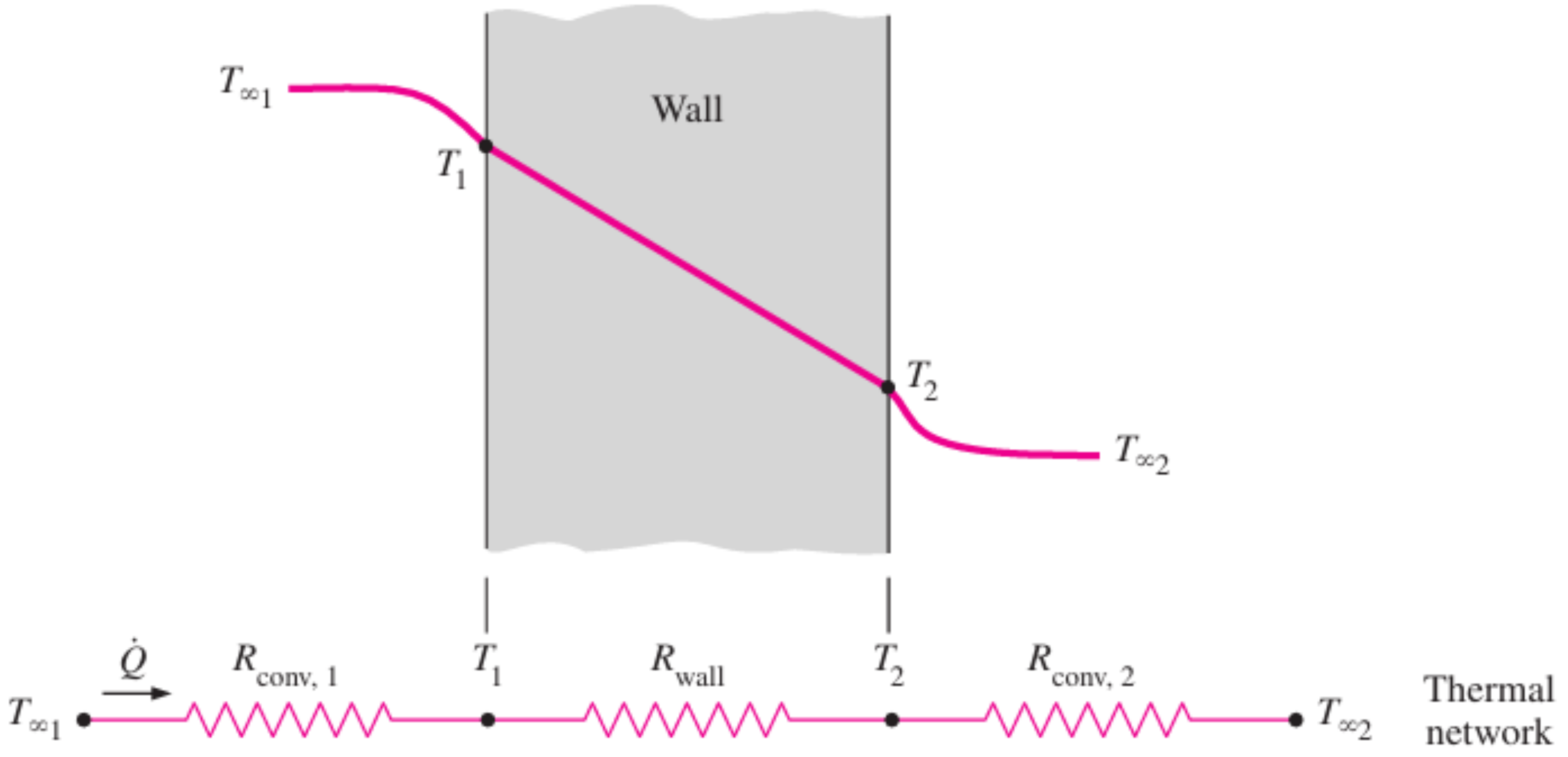
Where,

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \qquad \text{(K/W)}$$

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

$\boldsymbol{R_{rad}}$ is the thermal resistance of a surface against radiation, or the **radiation resistance** and
$\boldsymbol{h_{rad}}$ is the radiation heat transfer coefficient.

Thermal Resistance Concept



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,\,1} + R_{wall} + R_{conv,\,2}}$$

EXAMPLE 3–2

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of k = 0.78 W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be h_1 = 10 W/m2· °C and h_2 = 40 W/m^2· °C, which includes the effects of radiation.

EXAMPLE 3–2

Assumptions 1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

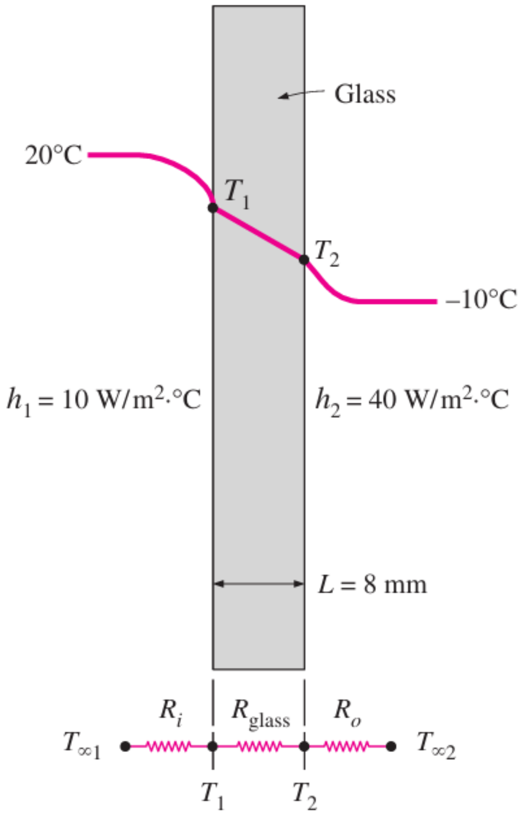
$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = \mathbf{266 \text{ W}}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \longrightarrow \begin{aligned} T_1 &= T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} \\ &= 20^\circ\text{C} - (266 \text{ W})(0.08333^\circ\text{C/W}) \\ &= \mathbf{-2.2^\circ C} \end{aligned}$$



HEAT CONDUCTION IN CYLINDERS

Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)} \qquad \textbf{(3-35)}$$

where $A = 2\pi rL$ is the heat transfer area at location r. Note that A depends on r, and thus it *varies* in the direction of heat transfer. Separating the variables in the above equation and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives

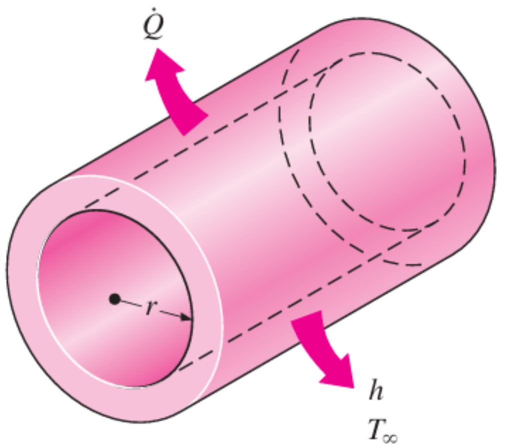
$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT \qquad \textbf{(3-36)}$$

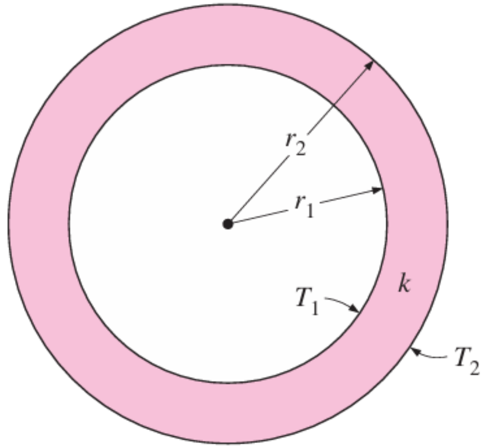
Substituting $A = 2\pi rL$ and performing the integrations give

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)} \qquad \textbf{(3-37)}$$

since $\dot{Q}_{\text{cond, cyl}} =$ constant. This equation can be rearranged as

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)} \qquad \textbf{(3-38)}$$

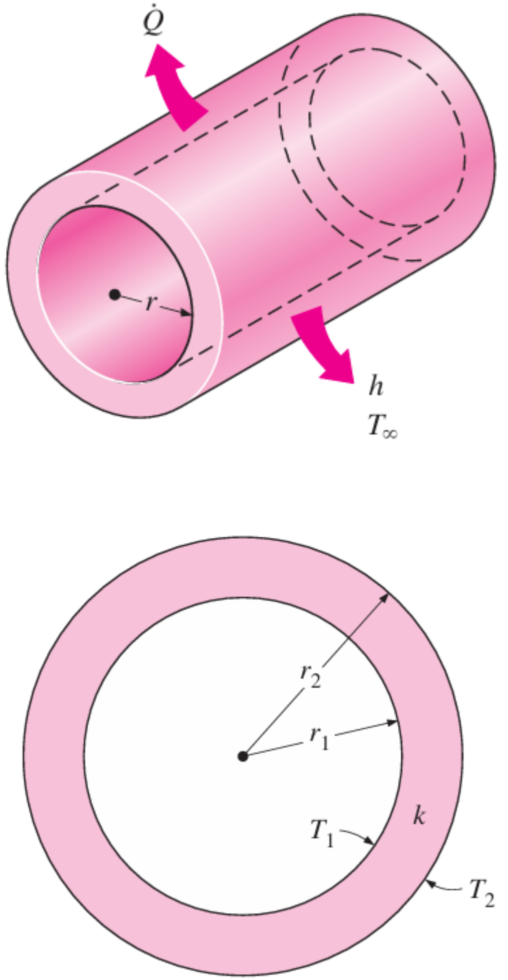




where

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})} \quad \textbf{(3-39)}$$

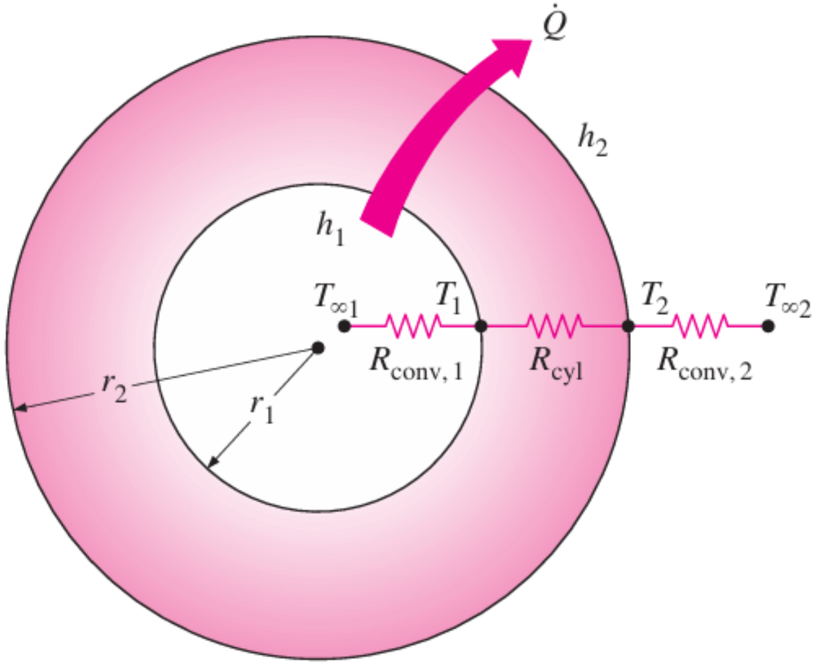
is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

where

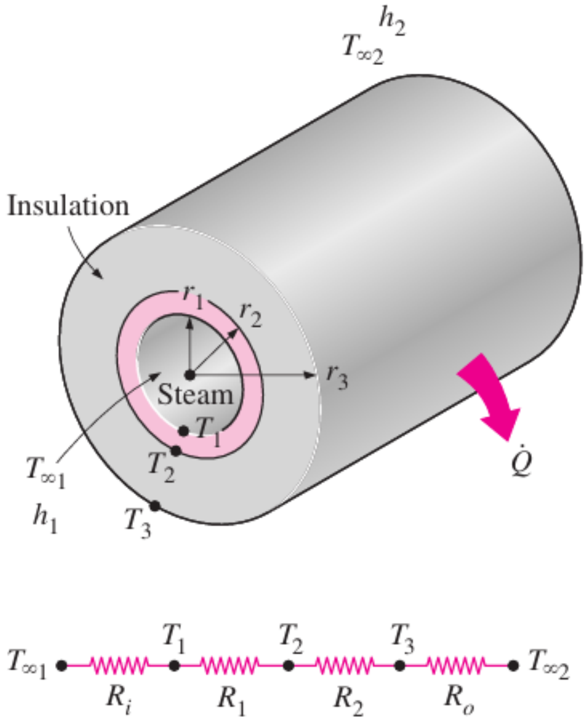
$$R_{total} = R_{conv,\,1} + R_{cyl} + R_{conv,\,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$



$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$

EXAMPLE 3–8

Steam at $T\infty_1$ = 320°C flows in a cast iron pipe (k = 80 W/m·°C) whose inner and outer diameters are D_1 = 5 cm and D_2 = 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with k = 0.05 W/m·°C. Heat is lost to the surroundings at $T\infty_2$ = 5°C by natural convection and radiation, with a combined heat transfer coefficient of h_2 18 W/m²·°C. Taking the heat transfer coefficient inside the pipe to be h_1 = 60 W/m²·°C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.



SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60 \text{ W/m}^2 \cdot °\text{C})(0.157 \text{ m}^2)} = 0.106°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 0.0002°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 2.35°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot °\text{C})(0.361 \text{ m}^2)} = 0.154°\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

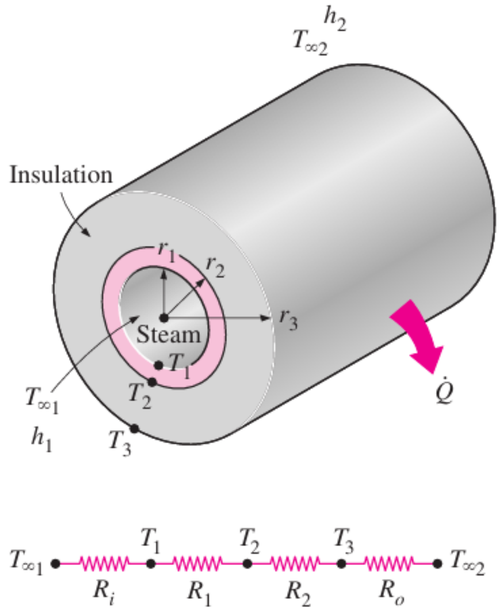
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)°\text{C}}{2.61°\text{C/W}} = \mathbf{121 \text{ W}} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002°\text{C/W}) = \mathbf{0.02°C}$$
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35°\text{C/W}) = \mathbf{284°C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.



CRITICAL RADIUS OF INSULATION

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant (Fig. 3–30). The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 3–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \qquad \textbf{(3-49)}$$

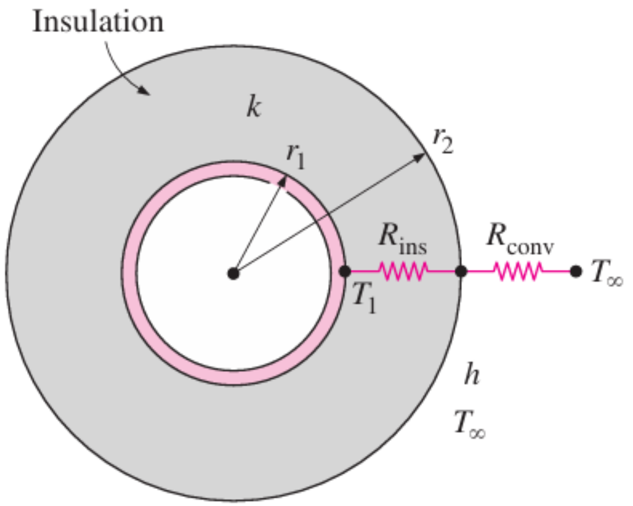


FIGURE 3–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

CRITICAL RADIUS OF INSULATION

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 3–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)} \tag{3-50}$$

Note that the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{\text{cr}}$, reaches a maximum when $r_2 = r_{\text{cr}}$, and starts to decrease for $r_2 > r_{\text{cr}}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{\text{cr}}$.

The important question to answer at this point is whether we need to be concerned about the critical radius of insulation when insulating hot water pipes or even hot water tanks.

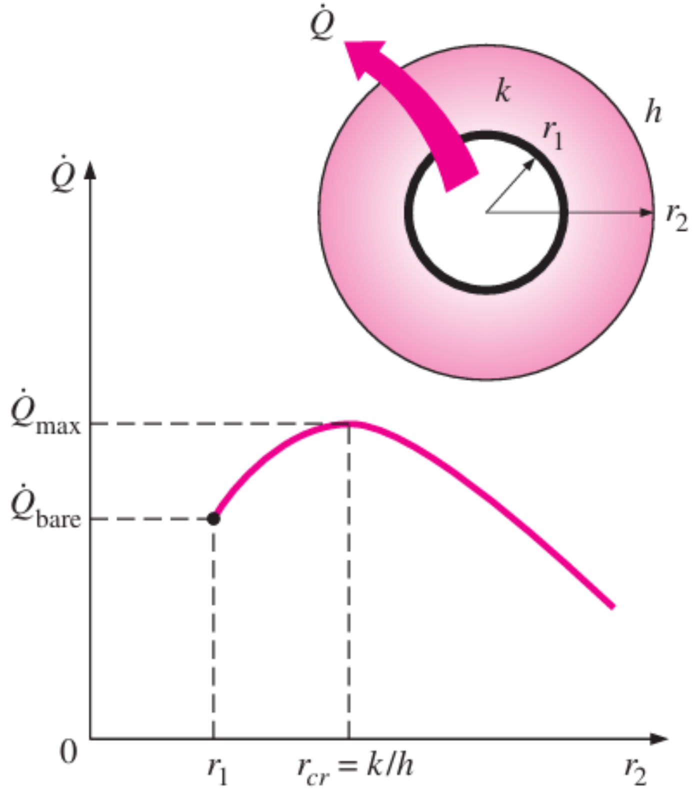
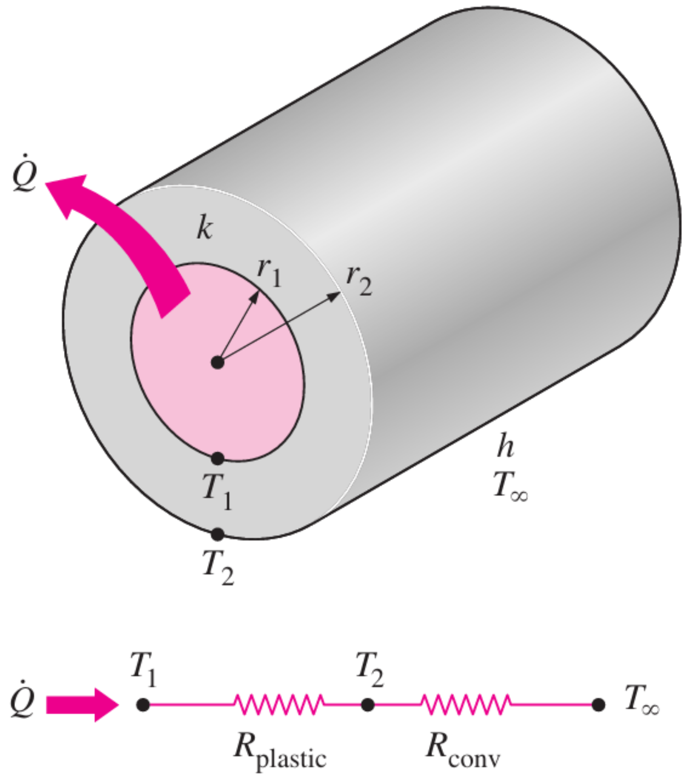


FIGURE 3–31

EXAMPLE 3–9

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm thick plastic cover whose thermal conductivity is k = 0.15 W/m·°C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T = 30°C with a heat transfer coefficient of h = 12 W/m²·°C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.



SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m · °C.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

Then the interface temperature can be determined from

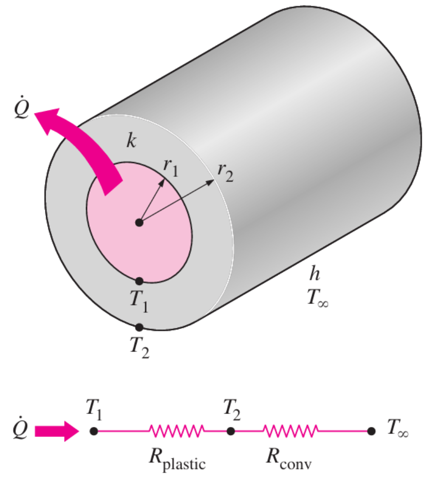
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}}$$
$$= 30^\circ\text{C} + (80\text{ W})(0.94^\circ\text{C/W}) = \mathbf{105^\circ C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot{}^\circ\text{C}}{12\text{ W/m}^2\cdot{}^\circ\text{C}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.



Thank You